Public Goods, Corruption, and the Political Resource Curse

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Abstract

Does corruption increase or decrease with public goods spending? I develop a model of public good provision by a rent-seeking political agent who can obtain rents by stealing government revenues and/or extracting bribes from the private sector in exchange for public goods. I show there is a threshold level of revenues below which the agent cannot steal, and therefore can obtain rents only from bribes. Higher revenues unambiguously increase public spending because nothing is stolen, and can decrease corruption (in the form of bribes) if the marginal social value of the public goods from which the bribes are extracted is sufficiently high. Above this threshold, the agent can increase her rents by stealing revenues. Revenues have no effect on public spending, but unambiguously increases corruption (in the form of theft). Hence, a political resource curse always emerges when resources provide politicians with 'too much' revenues - that is, beyond a threshold level that she could credibly spend on public goods.

Keywords: public goods, corruption, theft of public funds, bribery, political resource curse

JEL Codes: D73, H2, H41

1 Introduction

Corruption and the underprovision of public goods are two of the most formidable problems in the developing world. Many papers find that they are often bound together, but the nature of their relationship remains unresolved. Does corruption increase or decrease with public goods spending? Some cross-country evidence show that corruption is positively associated with some types of government expenditure, but negatively for other types of government spending.[1] The idea is that spending e.g. on military contracts and public works, generate large bribes and kickbacks, while other spending, e.g. on health, education, social services, do not. Arvate et al. (2010) and Hessami (2014), however, show that the positive association exists for most types of government expenditures, even across OECD countries. Micro-level evidence also demonstrate that public good provision provides opportunities for corruption by measuring leakages from public projects.[2]

The political resource curse literature suggests that corruption only occurs when the revenues that fund public good provision come from windfall gains like oil and natural-resource revenues.[3] Yet even a cursory look at cross-country data suggests the opposite. Figure 1 shows that while, overall, the incidence of bribery increases with military spending, such association is only apparent for countries with little reliance on oil revenues. In fact, for countries whose oil revenues are greater than 10 percent of GDP, the association disappears.

Empirical evidence is inconclusive because the theory governing the relationship between corruption and public good provision remains underdeveloped. On the one hand, canonical models of the rent-seeking political agent (e.g. Barro (1973), Ferejohn (1986), Persson and Tabellini (2000), Bueno de Mesquita et al. (1999, 2003, 2010)) show that public good provision is associated with a decrease in corruption. In these models, the agent can either spend government revenues on public goods, which benefit all citizens, or appropriate it for her own consumption and/or to buy political support. Corruption is thus tantamount to theft of government revenues and, in this sense, the agent is *revenue-seeking*. When the agent is revenue-seeking, public spending and corruption necessarily move in opposite directions, as more spending simply leaves less revenues for the agents' private use/consumption.

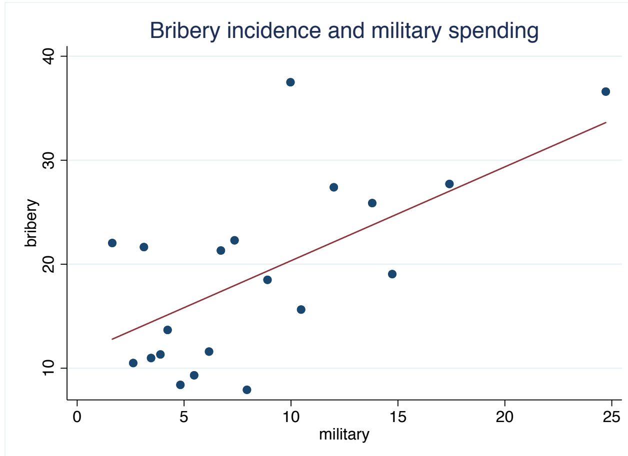
On the other hand, the agent might want to increase spending in order to obtain rents — that is, she could be *expenditure-seeking*. This type of rent-seeking is captured in the common agency models of bribery, pioneered by Bernheim and Whinston (1986a, 1986b), Dixit, Grossman, and Helpman (1997), and Grossman and Helpman (1994, 2001), in which principals from the private

[1]See Mauro (1998), Tanzi and Davoodi (1997, 2001), Gupta, Davoodi and Tiongson (2001), and Gupta, de Melo and Sharan (2001).

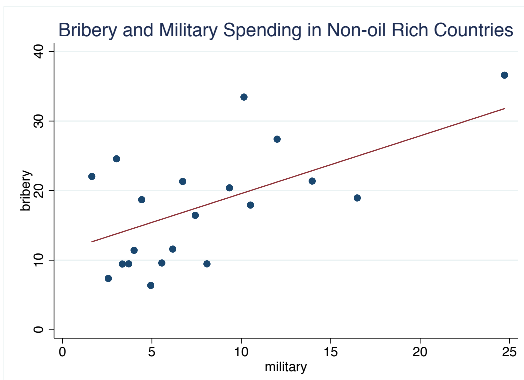
[2]See, e.g., Olken (2006, 2007), Reinikka and Svensson (2004), and Niehaus and Sukhtankar (2013). Olken and Pande (2012) provide a survey.

[3]See Ross (2015) for a survey of the empirical literature and Desierto (2018) for a survey of formal models.

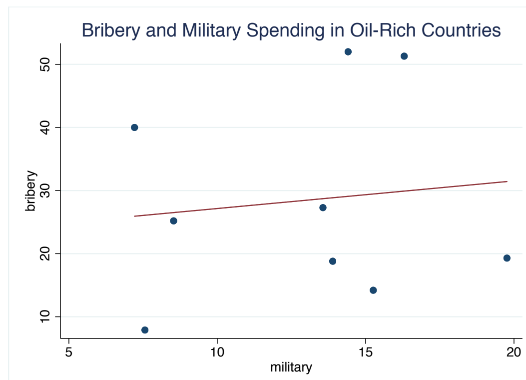
Figure 1: Does Corruption Increase with Military Spending?



(a) All available World Bank country-level data between 1997-2012



(b) Countries with oil revenues less than 10% of GDP



(c) Countries with oil revenues greater than 10% of GDP

Data used are from a pooled cross-section of countries for which some World Development Indicators are available between years 1997 to 2012 — specifically: *bribery*, which is the percentage of firms experiencing at least one bribe payment request; *military*, which is military expenditure as a percentage of GDP; and *oil*, which is oil rents as a percentage of GDP

sector offer bribes to their common political agent in exchange for their preferred policy, e.g. higher public-good spending.

When the agent may be both revenue– and expenditure-seeking, the relationship between public-good spending and corruption is therefore ambiguous. To the best of my knowledge, the political economy literature has not yet formally analyzed the case where the agent can obtain rents both from the theft of government revenues and from bribe payments.

I develop a model of public good provision from which two types of corruption can emanate —- theft of government revenues and bribery. My theoretical framework builds on the work of Grossman and Helpman (2001) who apply the common agency model with complete information to the problem of the optimal allocation of government revenues between two sets of principals, one of which offers a menu of contributions or bribes to the government in order to influence the latter to spend relatively more revenues towards that principal. Their model assumes, however, that the agent spends all of the revenues and, thus, obtains rents only by receiving bribes. Since the bribes are given in exchange for spending, and higher spending also increases principals' utility, a straightforward result is that an increase in government revenues induces more spending.

However, when the agent can steal some/all of the revenues, the effect of increased revenues on spending is not obvious. The agent might be induced to spend all of the additional revenues in exchange for more bribes, but she might also want to keep them for herself. By incorporating theft into the model, I show that there is a threshold of spending that has to be maintained. If revenues are at or below this required spending, all revenues are spent and theft is not possible. In this case, all rents come from bribes, and increasing revenues up to the threshold unambiguously increases public spending. Above the threshold, the agent can now steal some/all of the extra revenues, that is, while meeting the spending requirement. In equilibrium, *all* revenues above the threshold are stolen — all additional rents come from theft. Thus, further increases in revenues beyond the threshold have no effect on spending, nor on bribes.

The intuition is that when revenues are sufficiently large such that the agent can now steal some of them, the amount of bribes offered by the principal in exchange for spending has to be such that the agent's utility would match what it would be if the agent were to steal the revenues instead. For a given amount of revenues, the principal offers a bribe in exchange for a relatively higher share of those revenues — it compensates the agent for the loss in social welfare from 'over-allocating' towards the bribing principal. Now, if theft were also possible, then the bribes would not only have to pay for a higher share in the revenues, but they also have to compensate the agent for foregoing the theft of such revenues. Thus, the principal's marginal cost of preventing theft is larger than the agent's marginal benefit from stealing.

The structure of the remainder of the paper is as follows. The next section formally derives results, analyzes the implications on social welfare, and interprets the political resource curse in the light of the results. In section 3, I explicitly show that the revenue– and expenditure-seeking behavior of the agent occurs even when she can be made accountable to her principals through elections — such political accountability is imperfect because the rents from office can be used to influence electoral outcomes. Section 4 concludes with a summary of the contributions of the model.

2 The Model

Let T be government revenues allotted to total public spending $g_1 + g_2$. Principal 1 derives gross benefit $V(g_1)$, while principal 2 derives benefit $V(g_2)$, with $V'(\cdot) > 0, V''(\cdot) < 0$.

Principal 1 offers the government bribe b in exchange for g_1. Its net benefit from public spending is thus $V(g_1) - b$. The government then chooses g_1. It values rents, but also cares about social welfare. Now let total rents R include both bribes and unspent revenues (which the government steals), i.e. $R = T - g_1 - g_2 + b$. The government's utility is thus given by $U = \lambda[V(g_1) + V(g_2)] + (1 - \lambda)(T - g_1 - g_2 + b)$, where $\lambda \in (0, 1)$ is the weight it attaches to social welfare.

An equilibrium allocation is jointly efficient for the agent (government) and the principal who offers the bribe. That is, it is obtained by solving

$$\begin{aligned} &\max_{g_1, g_2, b} V(g_1) - b \\ s.t.\ &\lambda[V(g_1) + V(g_2)] + (1 - \lambda)(T - g_1 - g_2 + b) \geq \overline{U} \quad (a) \\ &g_1 + g_2 - T \leq 0 \quad (b), \end{aligned} \tag{1}$$

where $\overline{U}$ is the government's reservation utility - what it would obtain if it rejects principal 1's offer. Constraint (a) requires that the government's utility when it accepts the bribe is at least as large as when it rejects it. The possibility of theft is captured by constraint (b) - if it is binding, i.e. $g_1 + g_2 = T$, then all revenues are spent and theft is not possible. If it is slack, then theft occurs, with the amount of stolen revenues equal to $T - g_1 - g_2$. I thus call constraint (b) the "no-theft constraint".

I first analyze the equilibrium in which the no-theft constraint binds and, thus, only bribery is the source of the agent's rents.

2.1 Bribery

If the no-theft constraint binds, then $g_2 = T - g_1$ and problem (1) becomes[4]

$$\begin{aligned} &\max_{g_1, b} V(g_1) - b \\ s.t.\ &\lambda[V(g_1) + V(T - g_1)] + (1 - \lambda)(b) \geq \overline{U} \end{aligned} \tag{2}$$

In equilibrium, the above constraint binds with equality, which allows one to obtain the following expression for b:

$$b = \Big(\frac{1}{1 - \lambda}\Big)\big[\overline{U} - \lambda[V(g_1) + V(T - g_1)]\big], \tag{3}$$

which, when plugged into the maximand in (2), transforms (2) into the following unconstrained problem:

$$\max_{g_1} V(g_1) - \Big(\frac{1}{1 - \lambda}\Big)\big[\overline{U} - \lambda[V(g_1) + V(T - g_1)]\big]. \tag{4}$$

[4] This is the exact same problem in Grossman and Helpman (2001).

Equilibrium g_1^* thus satisfies the first-order condition (FOC) $F = V'(g_1^*) + \frac{\lambda}{1-\lambda}V'(g_1^*) - \frac{\lambda}{1-\lambda}V'(T - g_1^*) = 0$, or

$$V'(g_1^*) = \lambda V'(T - g_1^*). \tag{5}$$

That is, the equilibrium allocation is such that it attaches more weight to the marginal benefit from spending of the principal that offers a bribe, implying that $g_1^* > g_2^* = T - g_1^*$.

However, it does not necessarily follow that an increase in revenues induces a larger increase in g_1^* than in g_2^*. In particular:

Proposition 2.1. *Let* $x = \frac{V''(g_1^*)}{V''(T-g_1^*)}$. *Then:*
(a) if $\lambda > x$, *then* $\frac{dg_1^*}{dT} > \frac{dg_2^*}{dT} > 0$;
(b) if $\lambda < x$, *then* $\frac{dg_2^*}{dT} > \frac{dg_1^*}{dT} > 0$.

(All proofs are in appendix D.)

Note that revenues increase total spending since the government values social welfare to some extent, i.e. $\lambda > 0$. (See the proof of Proposition 2.1.) However, the effect on bribes is ambiguous. To compute the amount of bribes in equilibrium, note that $\overline{U}$ is the maximum utility that the government would obtain if bribes were zero. The government would then choose g_1^0 to maximize $\lambda[V(g_1) + V(T - g_1)]$, thereby achieving the social, i.e. first-best, optimum. In this case, $V'(g_1^0) = V'(T - g_1^0)$, which implies an equal allocation of T between sectors, that is, $g_1^0 = (T - g_1^0) = \frac{T}{2}$. Thus, $\overline{U} = \lambda[V(\frac{T}{2}) + V(\frac{T}{2})] = 2\lambda V(\frac{T}{2})$.

The equilibrium bribe is thus:

$$b^* = \frac{\lambda}{1-\lambda}\big[2V(\frac{T}{2}) - V(g_1^*) - V(T - g_1^*)\big]. \tag{6}$$

That is, the bribe compensates the agent for a fraction $\frac{\lambda}{1-\lambda}$ of the loss in social welfare.

The following result is obtained.

Proposition 2.2. *An increase in government revenues may increase or decrease corruption. Specifically, let* $y = \frac{V'(\frac{T}{2}) - V'(T-g_1^*)}{V'(g_1^*) - V'(T-g_1^*)}$. *Then:*
(a) if $\frac{dg_1^*}{dT} < y$, *then* $\frac{\partial b^*}{\partial T} > 0$.
(b) if $\frac{dg_1^*}{dT} > y$, *then* $\frac{\partial b^*}{\partial T} < 0$.
(c) if $\frac{dg_1^*}{dT} = y$, *then* $\frac{\partial b^*}{\partial T} = 0$.

That is, principal 1 would want to increase (decrease) the amount of the bribe if the agent would want to allocate the additional revenues towards principal 1 at a rate that is below (above) some threshold y. In turn, this threshold captures the marginal value of the spending on principal 1, relative to the spending on principal 2. (See the denominator of y.) Note that when the former is much larger than the latter, the threshold is smaller, which makes a decrease in corruption (a drop in b) more likely.

The following examples consider various functional forms for the principals' utility from public spending.

Running Example 1. Suppose $V(g_i) = \ln g_i$. Then $g_1^* = \frac{T}{1+\lambda}$, $g_2^* = \frac{\lambda T}{1+\lambda}$, and $\frac{dg_1^*}{dT} = \frac{1}{1+\lambda} > \frac{\lambda}{1+\lambda} = \frac{dg_2^*}{dT}$. Condition (a) of Proposition 2.1 is satisfied for all $\lambda \in (0,1)$, since x in this case is equal to λ^2. It can also be shown that $b^* = \frac{\lambda}{1-\lambda}[2\ln(\frac{T}{2}) - \ln(\frac{T}{1+\lambda}) - \ln(\frac{\lambda T}{1+\lambda})]$ and, thus, $\frac{\partial b^*}{\partial T} = 0$. Condition (c) of Proposition 2.2 is satisfied for all $\lambda \in (0,1)$, since y in this case is equal to $\frac{1}{1+\lambda}$.

Running Example 2. Suppose $V(g_i) = \sqrt{g_i}$. Then $g_1^* = \frac{T}{1+\lambda^2}$, $g_2^* = \frac{\lambda^2 T}{1+\lambda^2}$, and $\frac{dg_1^*}{dT} = \frac{1}{1+\lambda^2} > \frac{\lambda^2}{1+\lambda^2} = \frac{dg_2^*}{dT}$. Condition (a) of Proposition 2.1 is satisfied for all $\lambda \in (0,1)$, since x in this case is equal to λ^3. As for the equilibrium bribes, it can be shown that $b^* = \frac{\lambda}{1-\lambda}[2\sqrt{\frac{T}{2}} - \sqrt{\frac{T}{1+\lambda^2}} - \sqrt{\frac{\lambda^2 T}{1+\lambda^2}}]$, and that $\frac{\partial b^*}{\partial T} = \frac{\lambda}{1-\lambda}[\frac{1}{\sqrt{2T}} - \frac{(1+\lambda)}{2\sqrt{(1+\lambda^2)+T}}]$. Thus, it is now the case that $\frac{\partial b^*}{\partial T} < 0$, since $\frac{1}{\sqrt{2T}} < \frac{(1+\lambda)}{2\sqrt{(1+\lambda^2)+T}}$ or, simplifying, $\lambda < 1$. Condition (b) of Proposition 2.2 is satisfied for all $\lambda \in (0,1)$, since in this case, $\frac{dg_1^*}{dT} = \frac{1}{1+\lambda^2} > \frac{(1-\lambda)\sqrt{2}}{\sqrt{1+\lambda^2}} - \frac{1}{\lambda} - 1 = y$. To see this, one can simplify the latter inequality to $\frac{1}{\lambda} > (\sqrt{2(1+\lambda^2)} - \lambda)(1-\lambda)$ and note that the LHS is greater than 1, while the RHS is less than 1 for all $\lambda \in (0,1)$.

Running Example 3. Suppose $V(g_i) = -\frac{1}{g_i}$. Then $g_1^* = \frac{T}{1+\sqrt{\lambda}}$, $g_2^* = \frac{\sqrt{\lambda}T}{1+\sqrt{\lambda}}$, and $\frac{dg_1^*}{dT} = \frac{1}{1+\sqrt{\lambda}} > \frac{\sqrt{\lambda}}{1+\sqrt{\lambda}} = \frac{dg_2^*}{dT}$. Condition (a) of Proposition 2.1 is satisfied for all $\lambda \in (0,1)$, since x in this case is equal to $\lambda\sqrt{\lambda}$. Equilibrium bribe is $b^* = \frac{(1+\lambda)\sqrt{\lambda}-2\lambda}{(1-\lambda)T}$. Thus, in this case, $\frac{\partial b^*}{\partial T} = \frac{2\lambda-(1+\lambda)\sqrt{\lambda}}{(1-\lambda)T^2} > 0$, since $2\lambda > (1+\lambda)\sqrt{\lambda}$ for all $\lambda \in (0,1)$. (To see this, note that $2 > 1+\lambda$) and $\lambda > \sqrt{\lambda}$.) Condition (a) of Proposition 2.2 is satisfied since $\frac{dg_1^*}{dT} = \frac{1}{1+\sqrt{\lambda}} < \frac{4\lambda-(1+\sqrt{\lambda})^2}{(\lambda-1)(1+\sqrt{\lambda})^2} = y$, which simplifies to $2\lambda > (1+\lambda)\sqrt{\lambda}$.

2.2 Bribery and Theft

I now consider the case when the no-theft constraint is non-binding/slack, which implies that theft is now possible. Recall problem (1), in which $g_2 \neq T - g_1$:

$$\begin{aligned} &\max_{g_1,g_2,b} V(g_1) - b \\ s.t.\ &\lambda[V(g_1)+V(g_2)] + (1-\lambda)(T - g_1 - g_2 + b) \geq \overline{U} \quad (a) \\ &g_1 + g_2 - T \leq 0 \quad (b), \end{aligned} \tag{7}$$

and where (b) is the no-theft constraint. With constraint (a) holding with equality in equilibrium, the problem can be simplified into

$$\max_{g_1,g_2} V(g_1) - \frac{1}{1-\lambda}[\overline{U} - \lambda[V(g_1) + V(g_2)]] + T - g_1 - g_2 \\ s.t.\ g_1 + g_2 - T \leq 0 \tag{8}$$

To obtain the equilibrium allocation and total rents when both bribery and theft can occur, one needs to solve (8) for the case when the no-theft constraint is slack. In this case, the necessary conditions for optimal g_1^*, g_2^*, γ^* are given by the following Kuhn-Tucker conditions:

$$V'(g_1^*) + \frac{\lambda}{1-\lambda}V'(g_1^*) - 1 - \gamma^* = 0 \tag{9}$$

$$\frac{\lambda}{1-\lambda}V'(g_2^*) - 1 - \gamma^* = 0 \tag{10}$$

$$\gamma^*(g_1^* + g^* - T) = 0, \tag{11}$$

where γ is the Lagrange multiplier - the 'shadow price' of preventing theft.

The following results formally establish that not all revenues are stolen, and that some amount of spending is allocated to both principals. This also implies that bribes are non-zero. However, beyond this minimum spending, additional revenues have no effect on spending, nor on bribes - they are all stolen. Finally, I compare the effect of revenues on rents when the only source is bribery with the effect when both theft and bribery can occur. An increase in revenues can actually induce higher rents from the former than from the latter.

Proposition 2.3. *Even if theft occurs in equilibrium, some public spending are still allocated, i.e.* $g_1^*, g_2^* > 0$.

One can also solve for equilibrium bribe b^*. Constraint (a) in (7) implies that $b^* = \frac{1}{1-\lambda}[\overline{U} - \lambda[V(g_1^*) + V(g_2^*)]] + T - g_1^* - g_2^*$. To get the agent's reservation utility $\overline{U}$, note that if the agent rejects the bribe offer, she chooses g_1^0, g_2^0 from solving:

$$\max_{g_1,g_2} \lambda[V(g_1) + V(g_2)]] + (1-\lambda)(T - g_1 - g_2) \\ s.t.\ g_1 + g_2 - T \leq 0 \tag{12}$$

Necessary for g_1^0, g_2^0, γ^0 are the following Kuhn-Tucker conditions:

$$\lambda V'(g_1^0) - (1-\lambda) - \gamma^0 = 0 \tag{13}$$

$$\lambda V'(g_2^0) - (1-\lambda) - \gamma^0 = 0 \tag{14}$$

$$\gamma^0(g_1^0 + g_2^0 - T) = 0 \tag{15}$$

Thus, if the agent rejects the bribe offer, she obtains utility $\overline{U} = \lambda[V(g_1^0) + V(g_2^0)] + (1-\lambda)(T - g_1^0 - g_2^0)$, which means the equilibrium bribe when theft occurs is

$$b^* = \frac{\lambda}{1-\lambda}[V(g_1^0) + V(g_2^0) - V(g_1^*) - V(g_2^*)] - (g_1^0 + g_2^0 - g_1^* - g_2^*). \tag{16}$$

We can now generate the following comparative static results.

Proposition 2.4. *Changes in revenues have no effect on g_1^* or g_2^*, i.e.* $\frac{dg_1^*}{dT}, \frac{dg_2^*}{dT} = 0$.

One can obtain a result similar to Proposition 2.3 that establishes that $g_1^0, g_2^0 > 0$ (and $\gamma = 0$ - see appendix D). However:

Lemma 2.5. *Changes in revenues have no effect on dg_1^0 or dg_2^0, i.e.* $\frac{dg_1^0}{dT} = 0$ *and* $\frac{dg_2^0}{dT} = 0$.

Proposition 2.6. *Changes in revenues have no effect on the equilibrium bribe, i.e.* $\frac{\partial b^*}{\partial T} = 0$.

Running Example 1. With theft, $\gamma^* = 0$ and, thus, from equation (9), $g_1^* = \frac{1}{1-\lambda}$, and from equation (10), $g_2^* = \frac{\lambda}{1-\lambda}$. Thus, $\frac{dg_1^*}{dT} = \frac{dg_2^*}{dT} = 0$, which is consistent with Proposition 2.4. It can also be shown that $b^* = \frac{\lambda}{1-\lambda}[\ln(\frac{\lambda}{1-\lambda}) - \ln(\frac{1}{1-\lambda})] + 1$ and, thus, $\frac{\partial b^*}{\partial T} = 0$, which is consistent with Proposition 2.6.

Running Example 2. In this case, $g_1^* = \frac{1}{4(1-\lambda)^2}$, $g_2^* = \frac{\lambda^2}{4(1-\lambda)^2}$, and $\frac{dg_1^*}{dT} = \frac{dg_2^*}{dT} = 0$, which is consistent with Proposition 2.4. Equilibrium bribe is $b^* = \frac{\lambda}{1-\lambda}[\frac{1}{\lambda-1} - \frac{1+\lambda}{2(1-\lambda)}] - \frac{1}{2(\lambda-1)^2} + \frac{1+\lambda^2}{4(1-\lambda)^2}$ and, thus, $\frac{\partial b^*}{\partial T} = 0$, which is consistent with Proposition 2.6.

Running Example 3. In this case, $g_1^* = \frac{1}{\sqrt{1-\lambda}}$, $g_2^* = \frac{\sqrt{\lambda}}{\sqrt{1-\lambda}}$, and $\frac{dg_1^*}{dT} = \frac{dg_2^*}{dT} = 0$, which is consistent with Proposition 2.4. Equilibrium bribe is $b^* = \frac{\lambda}{1-\lambda}[\sqrt{1-\lambda} - \frac{\sqrt{1-\lambda}}{\sqrt{\lambda}}] + \frac{1-\sqrt{\lambda}}{\sqrt{1-\lambda}}$ and, thus, $\frac{\partial b^*}{\partial T} = 0$, which is consistent with Proposition 2.6.

Corollary 2.7. *Any additional revenues are stolen, and* $\frac{\partial R^*}{\partial T} = 1$.

Proposition 2.8. *Denote total corruption when theft does not occur as $R^{\not T}$ and R^* when theft occurs, in equilibrium, and re-label equilibrium g_1^* and g_2^* obtained in the case of no theft as $g_1^{\not T}$ and*

$g_2^{\mathcal{T}}$. *Let* $z = \frac{V'(\frac{T}{2}) - V'(T - g_1^{\mathcal{T}}) - (\frac{1-\lambda}{\lambda})}{V'(g_1^{\mathcal{T}}) - V'(T - g_1^{\mathcal{T}})}$. *Then:*

(a) if $\frac{dg_1^{\mathcal{T}}}{dT} < z$, *then* $\frac{\partial R^{\mathcal{T}}}{\partial T} > \frac{\partial R^*}{\partial T}$.

(b) if $\frac{dg_1^{\mathcal{T}}}{dT} > z$, *then* $\frac{\partial R^{\mathcal{T}}}{\partial T} < \frac{\partial R^*}{\partial T}$.

(c) if $\frac{dg_1^{\mathcal{T}}}{dT} = z$, *then* $\frac{\partial R^{\mathcal{T}}}{\partial T} = \frac{\partial R^*}{\partial T}$.

Running Example 1. Recall from section 2.1 that $\frac{\partial b^*}{\partial T} = \frac{\partial R^{\mathcal{T}}}{\partial T} = 0$ when $V(g_i) = \ln g_i$, and is thus less than $\frac{\partial R^*}{\partial T} = 1$. This is consistent with condition (b) of Proposition 2.8, since in this case, $z = \frac{1}{1+\lambda} + \frac{T}{\lambda - 1}$, which is less than $\frac{dg_1^{\mathcal{T}}}{dT} = \frac{1}{1+\lambda}$.

Running Example 2. Recall from section 2.1 that $\frac{\partial b^*}{\partial T} < 0$. Thus, $\frac{\partial R^{\mathcal{T}}}{\partial T} < \frac{\partial R^*}{\partial T}$. This is consistent with condition (b) of Proposition 2.8 - note that $\frac{dg_1^{\mathcal{T}}}{dT} = \frac{1}{1+\lambda^2} > \frac{\lambda\sqrt{2} - \sqrt{\lambda^2+1} - 2\sqrt{T}(1-\lambda)}{\sqrt{\lambda^2+1}(\lambda-1)} = z$. To see this, note that the inequality can be reduced to $\frac{2T(1+\lambda^2) + \sqrt{\lambda^2+1}}{1+\lambda^2} > \frac{\lambda\sqrt{2} - \sqrt{\lambda^2+1}}{\lambda - 1}$, and it can be shown that the LHS of this inequality is greater than 1, while the RHS is less than 1.

Running Example 3. Recall from section 2.1 that $\frac{\partial b^*}{\partial T} = \frac{2\lambda - (\lambda+1)\sqrt{\lambda}}{(1-\lambda)T^2} > 0$. Note that $\frac{2\lambda - (\lambda+1)\sqrt{\lambda}}{(1-\lambda)T^2} < 1$ if, simplifying, $\lambda < \frac{T^2 + \sqrt{\lambda}}{T^2 + 2 - \sqrt{\lambda}} = \overline{\lambda}$. Thus, $\frac{\partial R^{\mathcal{T}}}{\partial T} < \frac{\partial R^*}{\partial T}$ if $\lambda < \overline{\lambda}$, while $\frac{\partial R^{\mathcal{T}}}{\partial T} \geq \frac{\partial R^*}{\partial T}$ if $\lambda \geq \overline{\lambda}$. These are consistent with the conditions of Proposition 2.8 - for instance, one can set $\frac{dg_1^{\mathcal{T}}}{dT} = \frac{1}{1+\sqrt{\lambda}} > \frac{4\lambda - (1+\sqrt{\lambda}^2 - T^2(1-\lambda)}{(1+\sqrt{\lambda})^2(\lambda-1)} = z$ to capture condition (b), which precisely reduces to $\lambda < \frac{T^2+\sqrt{\lambda}}{T^2+2-\sqrt{\lambda}} = \overline{\lambda}$. Conditions (a) and (c) easily follow.

Figure 2 depicts the effect of government revenues on public-good spending and corruption, in which $S = g_1 + g_2$ is total spending, and subscripts 1 and 2 denote two particular examples. Propositions 2.3 and 2.4 imply that there is some threshold amount of revenues $\overline{T}$ at which total spending is positive, but beyond which spending does not increase further. Below the threshold, the no-theft constraint binds and, thus, all revenues are spent. Total public-good spending S thus has slope equal to 1 below $\overline{T}$ and 0 thereafter. When the no-theft constraint binds, Proposition 2.2 shows that bribes may increase or decrease. Consider, then, bribe curves b_1 and b_2 which show different possibilities below $\overline{T}$ but which, beyond $\overline{T}$, have slope 0 as implied by Proposition 2.6. Lastly, total rents below $\overline{T}$ come solely from bribes, in which case R_1 and R_2 coincide with b_1 and b_2, respectively. Above $\overline{T}$, rents come from both bribes and theft, with the amount of bribes fixed at b_1 and b_2, and all additional revenues are stolen. Thus, above $\overline{T}$, R_1 and R_2 have as their y-intercepts their respective intersections with b_1 and b_2, and slope equal to 1.

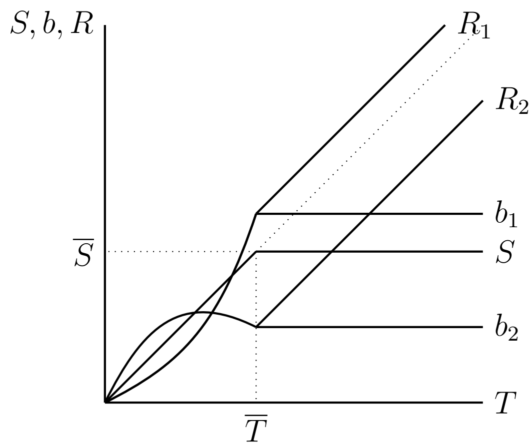


Figure 2: Effect of revenues on public goods and corruption

2.3 Social Welfare Loss from Theft and Bribery

Propositions 2.3, 2.4, Lemma 2.5, and Proposition 2.6 together imply that there is some required threshold amount of spending which generates a level of bribes, neither of which change when revenues increase further. As Corollary 2.7 shows, all additional corruption beyond this level is in the form of theft. Given the existence of such a threshold, one can then deduce that if actual revenues were at or below this threshold, the government is constrained to use all of these revenues to meet, to the extent possible, the threshold amount of spending. Thus, no theft is possible and only bribery occurs, in which case Proposition 2.1 shows that an increase in revenues unambiguously increases spending. Note that since the government also cares about social welfare, it would increase spending to all principals, even if only one principal would bribe. As a result, Proposition 2.2 shows that if the agent would not sufficiently increase spending towards the bribing principal, the latter will have to bribe the agent more aggressively. Thus, the bribe payment increases.

The two sources of corruption in the model - bribery and theft, produce two kinds of social welfare losses. Bribery induces an inter-sectoral misallocation of total revenues since it buys the bribing principal a higher share in total revenues. Note that social welfare does not include the utility that the agent derives from any rents she captures. Thus, while bribery is efficient between the bribing principal and the agent, it is *socially* inefficient. The equilibrium amount of bribes precisely compensates the agent for the loss in social welfare. Any decrease in bribes would thus increase social welfare. Meanwhile, theft induces the agent to underspend, since it keeps some revenues for herself. Theft is socially inefficient because spending to both sectors can be increased up to the exact amount of revenues, without making either of them worse off.

In an equilibrium in which the no-theft constraint binds, the only source of corruption is bribery, which implies that the only type of social inefficiency is the misallocation of revenues between the principals/sectors. In contrast, in an equilibrium in which the no-theft constraint is slack, both bribery and theft occur, but where bribery is limited to some threshold, above which all additional rents come from theft. Thus, in this case, there are welfare losses both from the misallocation and underspending of revenues.

One could also compare the losses from such equilibria to the off-equilibrium cases in which the agent rejects the bribe, in which case no bribery occurs. The following four cases thus exhaust the different scenarios:

Case 1: Only Bribery, No Theft

When theft is not possible, but the government accepts bribes, there is no underspending of revenues, but a misallocation thereof, in which the principal/sector that bribes receives a higher share. Recall (from equation (5)) that the equilibrium allocation is given by the FOC: $V'(g_1^*) = \lambda V'(T - g_1^*)$, where $\lambda < 1$, which implies that $g_1^* > g_2^* = T - g_1^*$. Thus, even though principals derive the same marginal utility from public spending, principal 1 obtains a larger share. Note, however, that because $\lambda > 0$, the condition is *not* $V'(g_1^*) = 0$, which implies $g_1^* < T$. Otherwise, if λ were zero, problem (2) would yield FOC $V'(g_1^*) = 0$, which implies $g_1 = T$. Because the government also cares about social welfare, some spending also has to be allocated to the non-bribing sector. Thus, the bribe compensates the government for the loss in social welfare from 'over-allocating' to the bribing sector. In effect, the inefficiency in intersectional allocation is mitigated because $\lambda > 0$.

Case 2: No Bribery, No Theft

If the government were to reject the bribe offer, recall that it would get its reservation utility $\lambda[V(g_1) + V(T - g_1)]$ (in which bribes are zero). It would then choose spending to maximize this utility, which would yield FOC $V'(g_1^0) = V'(T - g_1^0)$, implying that $g_1^0 = g_2^0 = T - g_1^0 = \frac{T}{2}$. That is, the agent would allocate revenues exactly according to the principals' marginal utilities from spending. Thus, if there were no corruption - no theft or bribery, inter-sectoral allocation is socially efficient. There is also no welfare loss from underspending, since $g_1^0 + g_2^0 = T$.

Case 3: Bribery and Theft

If theft were now possible, and the government also accepts bribes, then the Kuhn-Tucker conditions given by equations (9) to (11) imply that the allocation is such that $V'(g_1^*) = \lambda V'(g_2^*)$, where $g_1^* + g_2^* < T$. Again, there is intersectoral misallocation because $\lambda < 1$, but it is mitigated because λ is not equal to zero. In addition, there is also underspending of total revenues since $g_1^* + g_2^* < T$.

Case 4: No Bribery, Only Theft

If the government were to reject the bribes when theft is possible, its reservation utility includes the rents she would derive from stolen revenues. In this case, the Kuhn-Tucker conditions given by equations (13) to (15) imply that the the government would choose an allocation such that

$V'(g_1^0) = V'(g_2^0)$, where $g_1^0 + g_2^0 < T$. Inter-sectoral allocation is now socially efficient, but there is underspending of revenues.

Case 1 shows that the equilibrium when the no-theft constraint is binding generates welfare loss only from the misallocation of revenues, while case 3 shows that the equilibrium when the no-theft constraint is slack produces losses from both the misallocation and underspending of revenues. Since the no-theft constraint is slack when actual revenues are above some threshold, one may be tempted to infer that higher revenues always generate higher losses (as there would be two sources of welfare losses once revenues are above the threshold). However, this is not the case. I show in appendix A that the amount of bribes when revenues are below the threshold may be sufficiently large, and the bribes when revenues are above the threshold sufficiently small, such that the welfare-loss differential from bribes may be greater than the differential from theft. As a result, the social welfare loss (from bribes) at revenues below the threshold may be larger than the losses (from bribes and theft) at revenues above the threshold. Thus, it is not always true that losses are larger at higher levels of revenues.[5]

Why is it that only cases 1 and 3 can be supported in any equilibrium of the model? If we allow for the possibility of bribery, the bribing principal can now match the agent's (reservation) utility - what it would get if it rejected the bribe. That is, if the principal and agent can freely negotiate the amount of bribes, there is an amount that can induce the agent to accept the bribe. Thus, cases 2 and 4 in which bribes are rejected are off any equilibrium path. Note, however, that the extent to which the bribe can match the agent's reservation utility is limited. When revenues increase beyond some threshold, all additional rents now come from theft. This is because if the option to steal revenues now becomes viable, any additional bribe would not only have to compensate the agent for the loss in social welfare from the misallocation of revenues, but would also need to compensate the agent for foregoing the theft of revenues. This means that beyond the threshold, the (principal's) marginal cost of preventing theft is larger than the (agent's) marginal utility from theft. Thus, all revenues beyond the threshold are stolen.

While the *existence* of such a threshold is implied by the results, the value of the threshold cannot be determined in the model. One can solve for the equilibrium when the no-theft constraint binds, and when it is slack, but there is nothing that determines ex ante whether the constraint is binding or slack. One could indeed solve for $\overline{g_1}, \overline{g_2}$ when the constraint is binding, and let its sum $\overline{g_1} + \overline{g_2} = \overline{T}$ be the threshold amount of revenues above which theft is possible. That is, there is some minimum demand for spending that has to be met, and only when it is met can the agent steal the 'extra' revenues. Notice, however, that to obtain threshold $\overline{T}$, one has to assume that the

[5] Thus, notice from Figure 2 that there is a region below $\overline{T}$ (i.e. from the point of intersection between b_1 and b_2 until $\overline{T}$) in which bribes b_1 are larger than total rents from theft and bribes R_2 in a region above $\overline{T}$ (i.e. from $\overline{T}$ until the intersection of b_1 and R_2).

constraint is binding. Thus, the value of $\overline{T}$ cannot be (exogenously) determined - it is endogenous in the model.

2.4 An Application to the Political Resource Curse

The model thus demonstrates that the nature of the relationship between corruption and public-good spending depends on the existence of some threshold level of revenues. For an economy with government revenues above the threshold, the politician can keep public spending constant even while revenues are increasing. This implies that she can do so while remaining in office. Indeed, Section 3 explicitly shows that the rent-seeking politician can survive electoral competition because she can use the rents to buy votes. By this mechanism, the amount of public spending associated with the threshold level of revenues captures, as it were, the maximum value of public goods that is credibly demanded by the electorate. Beyond that level, their marginal utility from sharing in the politician's rents is greater than that from additional public goods.

When might government revenues exceed the threshold? I conjecture that an economy is more likely to be above the threshold the greater its reliance on revenues from oil, natural resources, and other windfall gains. A large influx of windfall income that flow directly to public coffers might be more easily captured through direct appropriation rather than indirectly by spending the windfall on public goods and extracting bribes therefrom. Thus, in equilibrium, political agents in an economy with considerable windfall revenues would be more likely to increase their rents as revenues increase by engaging in more theft, rather than more bribery.[6] In contrast, an economy that relies more heavily on tax revenues would be more likely to be below the threshold, in which case political agents can only keep extracting (bribe-)rents by spending more.

To show indirectly that such conjecture is plausible, Figure 3 plots military spending and corruption among countries that are reliant on oil revenues — those with oil revenues greater than 10 percent of GDP. In lieu of a measure for the theft of revenues, for which data are unavailable, panel (a) uses (the incidence of) bribery, while panel (b) proxies for bribery by using a measure of corruption that describes the lack of transparency in the public sector. There appears to be no association between military spending and bribery, which would be consistent with the model since the latter predicts that an economy above the threshold level of revenues would remain at a fixed level of public spending and, thus, of bribe-rents. That is, if political agents above the threshold increase their rents by stealing more (by keeping spending fixed) rather than taking more bribes from higher spending, then spending and bribery would be unrelated. Note that the same pattern

[6] This is indeed consistent with the formal literature on the political resource curse, where rent-seeking is modeled as theft or the appropriation of resource revenues. See Desierto (2018). The results here thus provide explicit justification for why corruption is more aptly modeled as theft, rather than bribe-taking, when depicting a political resource curse.

roughly holds when the extent of reliance on oil is increased to greater than 20 percent of total revenues (panels (c) and (d).) In contrast, Figure 4 plots military spending and bribery among countries with less reliance on oil revenues, and shows that the two variables increase together. If such countries were indeed below the threshold (and the marginal value of the public goods from which bribes are extracted is sufficiently low), the model predicts that both bribe-rents and public spending would increase with revenues, and would thus be positively associated.

Figure 5 confirms the patterns for countries that are more, and less, reliant on (windfall) revenues from minerals and from foreign aid.

The model can thus be used to explain the political resource curse. An increase in government revenues unambiguously increases corruption (in the form of theft) at the expense of public goods — that is, the political resource curse always exists, when the revenues exceed the true, 'credible' level of demand for public goods.

3 Political Accountability

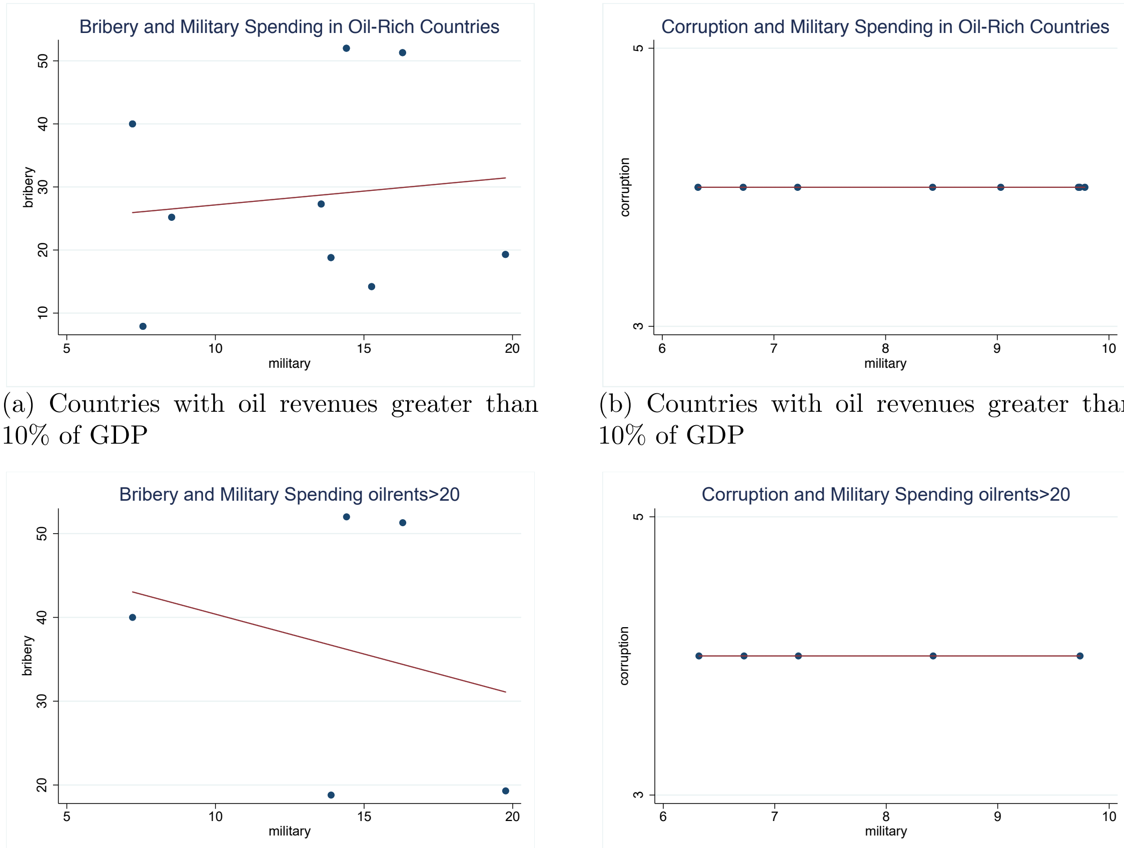
Thus far, it is assumed that the government values social welfare to some extent $\lambda \in (0, 1)$. This variable can capture institutional checks and balances that limit the extent of rent-seeking. However, it can also describe the extent of political competition which exerts pressure on the government to be more accountable to citizens. In this case, that $\lambda < 1$ implies that political accountability is imperfect. I show in this section that this is supportable if candidates in elections share their rents with the electorate by buying votes.[7]

Thus, following Grossman and Helpman, I now endogenize λ by modeling electoral competition. With two candidates competing in elections, the bribing principal offers bribes to each of them in exchange for a higher share in spending allocations once the candidate is in office. Each candidate can then use their respective bribes to buy votes. In equilibrium, the candidate that is more likely to win would allocate less spending to the bribing principal, which induces the latter to offer a bribe amount that is larger than what it offers to the other candidate. When candidates can also use stolen revenues as additional funds, the bribes to each candidate are such that they offset the differences in candidates' popularity relative to their ability to buy votes using stolen revenues.

To proceed with the formal analysis, let each principal $i = \{1, 2\}$ now explicitly consist of a group of individuals who choose the government by electing a party or candidate $k = \{A, B\}$. The game proceeds similarly, but with an additional last stage in which the government is selected.

[7]This is only one example. One can envisage situations in which politicians share rents through other forms of patronage. Neither is electoral competition the only form of (imperfect) political accountability that could support a rent-seeking equilibrium — a similar logic can operate under alternative forms of competition, e.g. via selectorate models in which the politician forms a coalition of supporters by offering to transfer some of her rents to coalition members.

Figure 3: Bribery and Spending Unrelated Above the Threshold



(a) Countries with oil revenues greater than 10% of GDP

(b) Countries with oil revenues greater than 10% of GDP

(c) Countries with oil revenues greater than 20% of GDP

(d) Countries with oil revenues greater than 20% of GDP

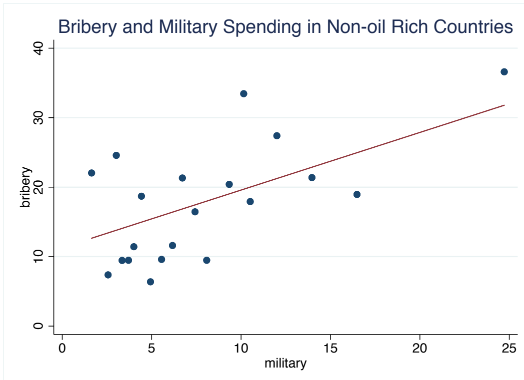
Data used are from a pooled cross-section of countries for which some World Development Indicators are available between years 1997 to 2012 — specifically: *bribery*, which is the percentage of firms experiencing at least one bribe payment request; *corruption*, which is the CPIA transparency, accountability, and corruption in the public sector rating (with 1 re-coded as most, and 6 least, transparent); *military*, which is military expenditure as a percentage of GDP; and *oil*, which is oil rents as a percentage of GDP

Specifically, the leader of group 1 offers bribe/contribution schedules to each candidate, who then announce her own policy. Each member of each group then vote for either candidate A or B.

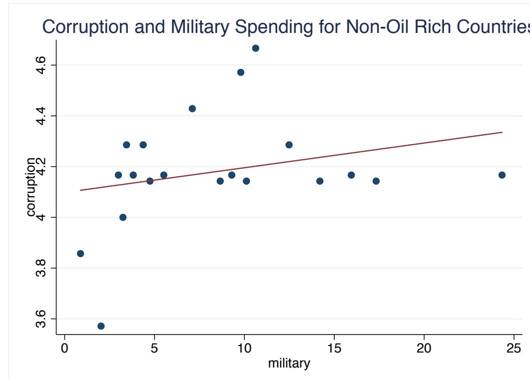
Suppose rents are now used by the candidates for campaign spending in order to sway some voters. That is, there is a fraction of total voters who are 'impressionable' in that they respond to such spending, while the rest are 'strategic' in that they vote only according to their policy preference. Again, I depart from Grossman and Helpman by allowing the possibility that rents can also come from stolen revenues.

I first derive the equilibrium under the assumption that the no-theft constraint binds, in which case only bribes can be used as campaign funds.

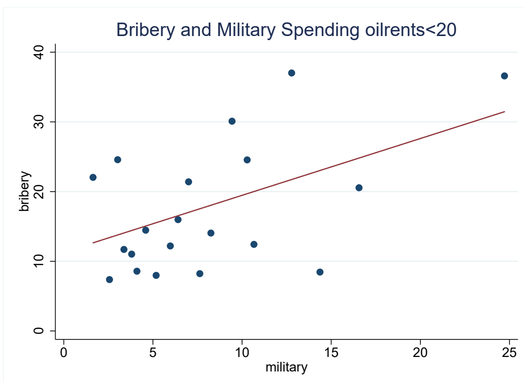
Figure 4: Bribery and Spending Positively Associated Below the Threshold



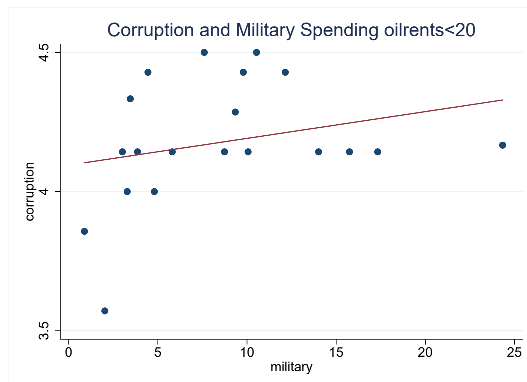
(a) Countries with oil revenues less than 10% of GDP



(b) Countries with oil revenues less than 10% of GDP



(c) Countries with oil revenues less than 20% of GDP



(d) Countries with oil revenues less than 20% of GDP

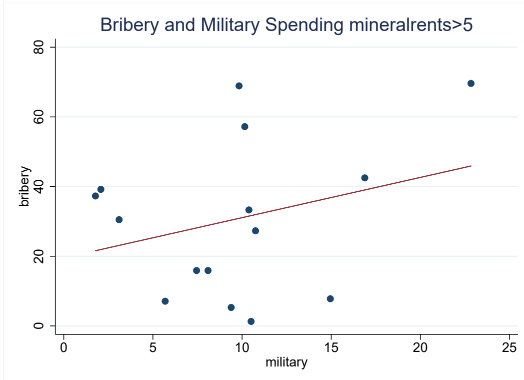
Data used are from a pooled cross-section of countries for which some World Development Indicators are available between years 1997 to 2012 — specifically: *bribery*, which is the percentage of firms experiencing at least one bribe payment request; *corruption*, which is the CPIA transparency, accountability, and corruption in the public sector rating (with 1 re-coded as most, and 6 least, transparent); *military*, which is military expenditure as a percentage of GDP; and *oil*, which is oil rents as a percentage of GDP

3.1 Bribery

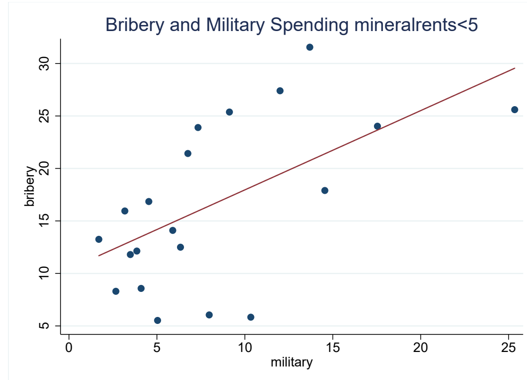
There are $N = N_1 + N_2$ voters, with $N_1 > 0$ voters belonging to group 1 and $N_2 > 0$ to group 2. As before, group 2 is the unorganized sector that is not capable of offering bribes, while group 1 can offer bribes. A voter can either be strategic or impressionable. A strategic voter j in group i has utility $V(g_i^k) + v_{ji}^k$, where $k = \{A, B\}$ indexes the candidate, with $V'(\cdot) > 0$, $V''(\cdot) < 0$. That is, $V(g_i^k)$ is the utility obtained from public spending to be allocated by k to group i, and is thus group-specific, whereas v_{ji}^k captures the voter's particular preference for k, and is thus voter-specific.[8]

[8]In Grossman and Helpman, the strategic voter's utility is $V_i(\mathbf{g}^k) + v_{ji}^k$, where $\mathbf{g}$ is the vector of policies, which in this case is $\mathbf{g} = (g_1, g_2)$. I simplify here by letting the first term be $V(g_i^k)$ - voter j only cares about the spending

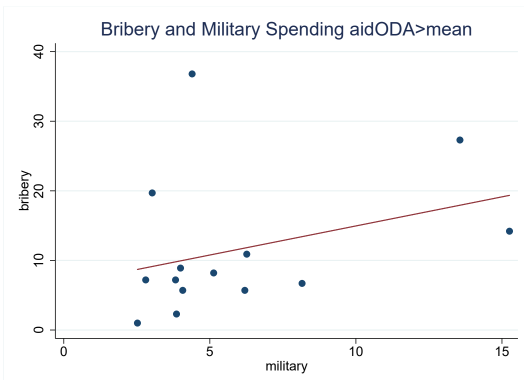
Figure 5: Positive Association Between Bribery and Spending More Apparent Below the Threshold



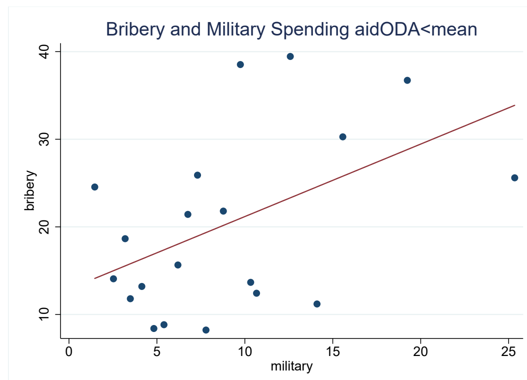
(a) Countries with mineral revenues greater than 5% of GDP



(b) Countries with mineral revenues less than 5% of GDP



(c) Countries receiving foreign aid greater than the mean amount in the sample



(d) Countries receiving foreign aid less than the mean amount in the sample

Data used are from a pooled cross-section of countries for which some World Development Indicators are available between years 1997 to 2012 — specifically: *bribery*, which is the percentage of firms experiencing at least one bribe payment request; *military*, which is military expenditure as a percentage of GDP; *oil*, which is oil rents as a percentage of GDP; *mineralrents*, which is mineral rents as a percentage of GDP; and *aidODA*, which is the net official development assistance and official aid received (in constant 2012 US$)

Let $v_{ji} = v_{ji}^B - v_{ji}^A$ denote the relative preference of voter j in group i for B over candidate A. For both groups, let v_{ji} be uniformly distributed, with mean b/f and density f. (Thus, b captures the relative popularity of candidate B.)

Strategic voter j in group i votes for A if and only if $v_{ji} \leq V(g_i^A) - V(g_i^B)$. This implies that the fraction of the strategic voters in group i who vote for A is:[9]

allocated to its own group, and by assuming that V takes the same functional form across groups.

[9]With mean b/f and density f, v_{ji} is uniformly distributed on the interval $\left[\frac{2b-1}{2f}, \frac{2b+1}{2f}\right]$. The share of strategic voters in group i who vote for A is thus $f[V(g_i^A) - V(g_i^B) - (\frac{2b-1}{2f})]$.

$$s_i^S = \frac{1}{2} - b + f[V(g_i^A) - V(g_i^B)] \tag{17}$$

(Thus, if both candidates adopt the same policy position, i.e. $g_i^A = g_i^B$, then $s_i^S = \frac{1}{2} - b$.) With $\sum N_i s_i^S = s^S N$, one can solve for the fraction of total strategic voters s^S who vote for A:

$$s^S = \frac{1}{2} - b + f\big[\frac{N_1}{N}[V(g_1^A) - V(g_1^B)] + \frac{N_2}{N}[V(g_2^A) - V(g_2^B)]\big]. \tag{18}$$

Now assume that for each group i, there is a fraction μ of strategic voters, and a fraction $1-\mu$ of impressionable voters who are influenced by campaign spending. Instead of specifying the explicit voting behavior of impressionable voters, I simply assume, as in Grossman and Helpman, that the fraction of impressionable voters in group i who vote for A when each candidate k spends bribes b^k on the campaign is:

$$s_i^I = \frac{1}{2} - b + e(b^A - b^B), \tag{19}$$

where e captures the effectiveness of such campaign spending. With s_i^I the same across groups, the vote share of A among all impressionable voters is thus

$$s^I = \frac{1}{2} - b + e(b^A - b^B). \tag{20}$$

Finally, with μ as the share of strategic, and $1-\mu$ the share of impressionable, voters, the overall share of votes for A is the weighted share

$$s = \frac{1}{2} - b + \mu f\big[\frac{N_1}{N}[V(g_1^A) - V(g_1^B)] + \frac{N_2}{N}[V(g_2^A) - V(g_2^B)]\big] + (1-\mu)e(b^A - b^B). \tag{21}$$

Each candidate wants to maximize its probability of winning. For A, this probability is the probability that $s > \frac{1}{2}$, which is greatest when her choice of g_i^A maximizes

$$U^A = \mu f[\frac{N_1}{N}V(g_1^A) + \frac{N_2}{N}V(g_2^A)] + (1-\mu)eb^A. \tag{22}$$

For B, the probability that $s < \frac{1}{2}$ (or the probability that she wins) is greatest when her choice g_i^B maximizes

$$U^B = \mu f[\frac{N_1}{N}V(g_1^B) + \frac{N_2}{N}V(g_2^B)] + (1-\mu)eb^B. \tag{23}$$

Notice that U^A and U^B are similar. More importantly, they are similar to the specification of the government's objective function U in section 2 - the weight λ that is attached to social welfare is now captured by parameters μ, f, e. Specifically, note that for the special case $\frac{N_1}{N} = \frac{N_2}{N} = \frac{1}{2}$, $U^k = \frac{\mu f}{2}[V(g_1^k) + V(g_2^k)] + (1-\mu)eb^k$, and if $e = \frac{f}{2}$, then $U^k = \mu[V(g_1^k) + V(g_2^k)] + (1-\mu)b^k$. In this case, the weight λ that the incumbent government attaches to social welfare in section 2 is simply motivated by the fraction μ of strategic voters.

Now recall that bribes are offered to the candidates by group 1. Group 1 thus has the problem of maximizing its members' expected benefit from g_1, net of the bribes it gives to candidates A and/or B. However, as before, it is constrained by the requirement that A and B each attain at least their reservation utilities $\overline{U^k}$, i.e. when bribes are zero. That is, the bribe offer has to at least compensate each candidate from adopting a level of spending g_i that is different from the level that maximizes the welfare of the average strategic voter.

Group 1 derives total benefit $N_1 V(g_1^k)$ if k is elected, and from its view, the ex-ante probability that A is elected is $F(\Delta)$, where

$$\Delta = U^A - U^B = \mu f\left[\frac{N_1}{N}[V(g_1^A) - V(g_1^B)] + \frac{N_2}{N}[V(g_2^A) - V(g_2^B)]\right] + (1-\mu)e(b^A - b^B). \tag{24}$$

Thus, group 1 solves:

$$\begin{aligned} &\max_{g_i^k, b^k} F(\Delta)N_1V(g_1^A) + (1 - F(\Delta))N_1V(g_1^B) - \sum_k b^k \\ &s.t.\ \mu f\left[\frac{N_1}{N}V(g_1^k) + \frac{N_2}{N}V(g_2^k)\right] + (1-\mu)eb^k \geq \overline{U^k}, \end{aligned} \tag{25}$$

for each $k = \{A, B\}$. Assuming that the constraints hold with equality, one can then obtain an expression for the bribe schedule that is offered to each candidate:[10]

$$b^k = \Big[\frac{1}{(1-\mu)e}\Big]\Big[\overline{U^k} - \mu f\Big[\frac{N_1}{N}V(g_1^k) + \frac{N_2}{N}V(g_2^k)\Big]\Big]. \tag{26}$$

Recall that when there are no theft of government revenues, $g_2 = T - g_1$. Using this fact and plugging in the expression for b^k into group 1's objective function, the group's problem can be re-cast as

$$\begin{aligned} &\max_{g_1^A, g_1^B} F(\Delta^T)N_1V(g_1^A) + (1 - F(\Delta^T))N_1V(g_1^B) \\ &- \Big[\frac{1}{(1-\mu)e}\Big]\Big[(\overline{U^A} + \overline{U^B}) - \mu f\Big[\frac{N_1}{N}(V(g_1^A) + V(g_1^B)) + \frac{N_2}{N}(V(T - g_1^A) + V(T - g_1^B))\Big]\Big], \end{aligned} \tag{27}$$

[10] In Grossman and Helpman, the case when the constraints hold with equality is interpreted as one in which the group has a pure 'influence' motive. That is, it offers bribes in order to influence policy. They also consider the case when the constraint is a strict inequality - in this case, the group also has an 'electoral' motive in that it gives more than what is necessary to influence policy, which can then be used for greater campaign spending. They show, however, that even with electoral motives, the qualitative results are the same - bribes are offered to both candidates.

where $\Delta^{\not T}$ is the same as equation (24), but now indexed by $\not T$ to distinguish this case as one in which theft is not possible. This yields the following FOCs:

$$V'(g_1^{A^*}) = \alpha^{A^*} V'(T - g_1^{A^*}) \tag{28}$$

$$V'(g_1^{B^*}) = \alpha^{B^*} V'(T - g_1^{B^*}), \tag{29}$$

where $\alpha^{A^*} = \frac{\mu f \frac{N_2}{N}}{\mu f \frac{N_1}{N} - x}$, $\alpha^{B^*} = \frac{\mu f \frac{N_2}{N}}{\mu f \frac{N_1}{N} + y}$, are the equilibrium 'weights' candidate A and B, respectively, attach to group 2's marginal utility from public spending, and where I have defined the following:[11] $x \equiv \frac{N_1 F(\Delta^{\not T})}{N_1 \frac{\partial F}{\partial \Delta^{\not T}}[V(g_1^{B^*}) - V(g_1^{A^*})] - [\frac{1}{(1-\mu)e}]}$ and $y \equiv \frac{N_1(1 - F(\Delta^{\not T}))}{N_1 \frac{\partial F}{\partial \Delta^{\not T}}[V(g_1^{B^*}) - V(g_1^{A^*})] + [\frac{1}{(1-\mu)e}]}$. Also, let $\mu f \frac{N_1}{N} \neq x$, $\mu f \frac{N_1}{N} \neq -y$.

I now characterize $g_1^{k^*}$ in several ways.

First, in equilibrium, both A and B offer not to spend all of revenues toward group 1, as each candidate attaches non-zero weight to the group 2's marginal utility from public goods. That is:

Proposition 3.1. *Each candidate $k = \{A, B\}$ offers $g_1^{k^*} < T$.*

Such offers, however, are not necessarily the same. If A's probability of being elected is high enough, the candidate offers less to group 1 than B does. Specifically:

Proposition 3.2. *Define threshold $z \equiv \frac{1}{2} - \frac{N_1 \frac{\partial F}{\partial \Delta^{\not T}}[V(g_1^{B^*}) - V(g_1^{A^*})]}{\frac{2}{(1-\mu)e}}$. If:*
(i)$F(\Delta^{\not T}) > z$, then $g_1^{A*} < g_1^{B*}$.
(ii)$F(\Delta^{\not T}) < z$, then $g_1^{A*} > g_1^{B*}$.
(iii)$F(\Delta^{\not T}) = z$, then $g_1^{A*} = g_1^{B*}$.

One might also be interested in comparing the candidates' public spending offers with the social optimum, that is, when bribes are rejected. To do so, I derive expressions for $\overline{U^k}$ by letting $b^k = 0$. In this case, $U^k = \mu f[\frac{N_1}{N} V(g_1^k) + \frac{N_2}{N} V(T - g_1^k)]$ which, when maximized, yields FOCs:

$$V'(g_1^{A^0}) = \frac{N_2}{N_1} V'(T - g_1^{A^0}) \tag{30}$$

$$V'(g_1^{B^0}) = \frac{N_2}{N_1} V'(T - g_1^{B^0}). \tag{31}$$

[11] See appendix B.

Notice, then, that both A and B would offer the same public spending if they reject bribes.[12] Such spending is socially optimal since it weighs the total marginal utilities of each group equally, i.e. $N_1 V'(g_1^{k^0}) = N_2 V'(T - g_1^{k^0})$. Henceforth, I omit the superscript k to denote the socially optimal spending for group 1 as g_1^0.

To establish the relative magnitudes of $g_1^{k^*}$ to g_1^0, the following Lemmas are useful.

Lemma 3.3. *Recall x and y. If:*
(i) $x > 0$, then $\alpha^{A^*} > \frac{N_2}{N_1}$;
(ii) $x < 0$, then $\alpha^{A^*} < \frac{N_2}{N_1}$;
(iii) $x = 0$, then $\alpha^{A^*} = \frac{N_2}{N_1}$;
(iv) $y > 0$, then $\alpha^{B^*} < \frac{N_2}{N_1}$;
(v) $y < 0$, then $\alpha^{B^*} > \frac{N_2}{N_1}$;
(vi) $y = 0$, then $\alpha^{B^*} = \frac{N_2}{N_1}$.

Lemma 3.4. *Define $w \equiv \frac{1}{(1-\mu)eN_1 \frac{\partial F}{\partial \Delta \not{T}}}$.*
(i) If $g_1^{A^*} = g_1^{B^*}$, then $x < 0, y > 0$;
(ii) If $g_1^{A^*} > g_1^{B^*}$, then $x < 0$, and
(ii.1) $y > 0$ if $V(g_1^{A^*}) - V(g_1^{B^*}) > w$;
(ii.2) $y < 0$ if $V(g_1^{A^*}) - V(g_1^{B^*}) < w$;
(iii) If $g_1^{A^*} < g_1^{B^*}$, then $y > 0$, and
(iii.1) $x > 0$ if $V(g_1^{B^*}) - V(g_1^{A^*}) > w$;
(iii.2) $x < 0$ if $V(g_1^{B^*}) - V(g_1^{A^*}) < w$.

It is then straightforward to obtain the following result.

Proposition 3.5. *Recall w and z.*
(i) If $F(\Delta \not{T}) = z$, then $g_1^{A^*} = g_1^{B^*} > g_1^0$.
(ii) If $F(\Delta \not{T}) < z$, then:
(ii.1) $g_1^{A^*} > g_1^{B^*} > g_1^0$ if $V(g_1^{A^*}) - V(g_1^{B^*}) > w$;
(ii.2) $g_1^{A^*} > g_1^0 > g_1^{B^*}$ if $V(g_1^{A^*}) - V(g_1^{B^*}) < w$;
(iii) If $F(\Delta \not{T}) > z$, then:

[12] Equations (30) and (31) imply that $\frac{V'(g_1^{A^0})}{V'(T-g_1^{A^0})} = \frac{V'(g_1^{B^0})}{V'(T-g_1^{B^0})}$. Suppose that $g_1^{A^0} > g_1^{B^0}$. Then $V'(g_1^{A^0}) > V'(g_1^{B^0})$, which requires that $V'(T-g_1^{A^0}) > V'(T-g_1^{B^0})$ and, in turn, that $(T-g_1^{A^0}) > (T-g_1^{B^0})$ or, re-arranging, $0 > g_1^{A^0} - g_1^{B^0}$. This contradicts $g_1^{A^0} > g_1^{B^0}$. One can derive an analogous contradiction for $g_1^{A^0} < g_1^{B^0}$. Thus, $g_1^{A^0} = g_1^{B^0}$, which implies that $(T - g_1^{A^0}) = (T - g_1^{B^0})$.

(iii.1) $g_1^{B^*} > g_1^{A^*} > g_1^0$ *if* $V(g_1^{B^*}) - V(g_1^{A^*}) > w$;
(iii.2) $g_1^{B^*} > g_1^0 > g_1^{A^*}$ *if* $V(g_1^{B^*}) - V(g_1^{A^*}) < w$;

Thus, by Proposition 3.5, at least one candidate always allocates to group 1 more than the socially optimal level. Notice from (ii) and (iii) that when group 1 is large, w is small, which makes it more likely that both A and B overspend on this group.

One last characterization of $g_1^{k^*}$ can be made by using the restriction that bribes are non-negative. Plugging $\overline{U^k} = \mu f[\frac{N_1}{N}V(g_1^0) + \frac{N_2}{N}V(T - g_1^0)]$ into equation (26) gives equilibrium bribe offer by k:

$$b^{k^*} = \Big[\frac{1}{(1-\mu)e}\Big]\Big[\mu f[\frac{N_1}{N}(V(g_1^{k^0}) - V(g_1^{k^*})) + \frac{N_2}{N}(V(T - g_1^{k^0}) - V(T - g_1^{k^*}))]\Big]. \tag{32}$$

If $b^{k^*} \geq 0$, it must be that $[V(g_1^{k^*}) - V(g_1^{k^0})] \leq \frac{N_2}{N_1}[V(T - g_1^{k^0}) - V(T - g_1^{k^*})]$. Thus:

Proposition 3.6. *The equilibrium allocation offered by candidate* $k = \{A, B\}$ *is such that, for each* k, $\frac{V(T-g_1^0)}{V(T-g_1^{k^*})} \geq \frac{\alpha^{k^*}N_1 + N_2}{2N_2}$.

Notice again that when group 1 is large, $g_1^{k^*}$ is large relative to g_1^0 (and, thus, $T - g_1^{k^*}$ is small relative to $T - g_1^{k^0}$), so that the LHS is large enough for the condition to hold.

It remains to compare the equilibrium bribe offers to A and B. I show that group 1 offers relatively more bribes to the candidate that would allocate less spending to the group.

Using (33), one can take the difference:

$$b^{A^*} - b^{B^*} = \Big[\frac{1}{(1-\mu)e}\Big]\mu f\Big[\frac{N_1}{N}[V(g_1^{B^*}) - V(g_1^{A^*})] + \frac{N_2}{N}[V(T - g_1^{B^*}) - V(T - g_1^{A^*})]\Big], \tag{33}$$

from which the following result is obtained.

Proposition 3.7. *If:*
(i) $g_1^{A^*} > g_1^{B^*}$, *then* $b^{A^*} < b^{B^*}$;
(ii) $g_1^{A^*} < g_1^{B^*}$, *then* $b^{A^*} > b^{B^*}$;
(iii) $g_1^{A^*} = g_1^{B^*}$, *then* $b^{A^*} = b^{B^*}$.

3.2 Bribery and Theft

I now consider the case when the no-theft constraint is slack, such that the candidate can also obtain rents from the theft of government revenues. To keep as close as possible to Grossman and Helpman, I let the agent use (anticipated) stolen revenues the way she uses the bribes from group 1, that is, to influence impressionable voters.[13] I show that with such additional campaign funds, it is still the case that when candidate A has a sufficiently high ex ante probability of winning, she can afford to offer lower spending to group 1, relative to candidate B's offer. However, both candidates now have the same pattern of spending allocation in that they both offer to either overspend on group 1, or on group 2, depending on the relative size of the groups. In the previous case when the only source of campaign funds is bribery and, hence, group 1, it is always the case that at least one candidate overspends on group 1. Now that rents from theft can also be used to buy votes, there is less dependence on group 1, which allows both candidates to cater more to group 2 as the latter becomes large.

As before, group 1 offers bribe amounts to A and B such that their differences are offset and their ex post probabilities of winning are the same. Note that when there is no theft of revenues, candidates offer to spend the same *total* amount, i.e. total revenues, and differ only in the proportion they allot between groups. Thus, group 1 offers a relatively higher bribe to the candidate that would spend relatively less on group 1. However, when revenues can be stolen, the candidates can also differ in how much they would steal and, thus, the total amount that they would spend. In this case, group 1 now has to consider that candidates, without bribes, can still have different ex ante capabilities of influencing impressionable voters since they can be bought by stolen revenues, weighed against their ability to satisfy strategic voters who are concerned with social welfare. I thus find that relatively more bribes are offered to the candidate whose 'theft' advantage over the other candidate and, hence, her relative ex ante advantage in influencing impressionable voters, is not enough to offset the relative advantage of the other candidate in satisfying strategic voters. The higher bribes precisely augment campaign funds such that these relative advantages cancel out.

To proceed, I now add $T - g_1^k - g_2^k$ to each candidate k's campaign funds such that the vote share of A among impressionable voters is $s^I = \frac{1}{2} - b + e(b^A - b^B + (T - g_1^A - g_2^A) - (T - g_1^B - g_2^B)) = \frac{1}{2} - b + e(b^A - b^B + g_1^B - g_1^A + g_2^B - g_2^A)$. The total vote share of A is now:

$$s = \frac{1}{2} - b + \mu f\Big[\frac{N_1}{N}[V(g_1^A) - V(g_1^B)] + \frac{N_2}{N}[V(g_2^A) - V(g_2^B)]\Big] + (1-\mu)e(b^A - b^B + g_1^B - g_1^A + g_2^B - g_2^A). \quad (34)$$

Thus, A and B's respective probability of winning are highest when the following are maximized:

$$U^A = \mu f[\frac{N_1}{N}V(g_1^A) + \frac{N_2}{N}V(g_2^A)] + (1-\mu)e(b^A - g_1^A - g_2^A) \quad (35)$$

$$U^B = \mu f[\frac{N_1}{N}V(g_1^B) + \frac{N_2}{N}V(g_2^B)] + (1-\mu)e(b^B - g_1^B - g_2^B), \quad (36)$$

[13]The fact that the actual theft occurs once the agent is in office is irrelevant. Candidates either advance the 'payment' to impressionable voters, or simply promise to pay them after the election. Note that the model similarly ignores the timing of the payment of bribes, as it only solves for the equilibrium bribe *offer*.

and the ex-ante probability that A is elected is $F(\Delta)$ where, now,

$$\Delta = U^A - U^B = \mu f\Big[\frac{N_1}{N}[V(g_1^A) - V(g_1^B)] + \frac{N_2}{N}[V(g_2^A) - V(g_2^B)]\Big] + (1-\mu)e(b^A - b^B - (g_1^A + g_2^A) + (g_1^B + g_2^B)). \quad (37)$$

Group 1 thus solves:

$$\begin{aligned} \max_{g_i^k, b^k} F(\Delta)N_1V(g_1^A) + (1 - F(\Delta))N_1V(g_1^B) - \sum_k b_T^k \\ s.t.\ \mu f[\frac{N_1}{N}V(g_1^k) + \frac{N_2}{N}V(g_2^k)] + (1-\mu)e(b_T^k - g_1^k - g_2^k) \geq \overline{U_T^k} \quad (a) \\ g_1^k + g_2^k \leq T \quad (b), \end{aligned} \quad (38)$$

for each $k = \{A, B\}$, and bribes b_T^k and the agent's reservation utility $\overline{U_T^k}$ are subscripted by T to distinguish the case when theft can occur.

Note that when constraint (b) is binding, Δ collapses back to equation (24), and the optimization problem reduces to (27) - the case of no theft, by letting $g_1^k = T - g_2^k$.

To see this, note that one gets the following expression for bribes by letting constraint (a) bind with equality:

$$b_T^k = \Big[\frac{1}{(1-\mu)e}\Big]\Big[\overline{U_T^k} - \mu f[\frac{N_1}{N}V(g_1^k) + \frac{N_2}{N}V(g_2^k)]\Big] + g_1^k + g_2^k, \quad (39)$$

where the reservation utilities are given by $\overline{U_T^k} = \mu f[\frac{N_1}{N}V(g_1^{k^0}) + \frac{N_2}{N}V(g_2^{k^0})] - (1-\mu)e(g_1^{k^0} + g_2^{k^0})$, i.e. when bribes are rejected. Plugging the expression in (39) into the maximand of (38), the problem then becomes:

$$\begin{aligned} \max_{g_1^k, g_2^k} F(\Delta)N_1V(g_1^A) + (1 - F(\Delta))N_1V(g_1^B) \\ -\Big[\frac{1}{(1-\mu)e}\Big]\Big[(\overline{U_T^A} + \overline{U_T^B}) - \mu f[\frac{N_1}{N}(V(g_1^A) + V(g_1^B)) + \frac{N_2}{N}(V(g_2^A) + V(g_2^B))]\Big] - (g_1^A + g_2^A) - (g_1^B + g_2^B), \\ s.t.\ g_1^A + g_2^A - T \leq 0;\ g_1^B + g_2^B - T \leq 0. \end{aligned} \quad (40)$$

The previous no-theft case is the special instance when the constraints bind, i.e. $g_1^k = T - g_2^k$, $\overline{U_T^k}$ becomes $\overline{U^k} = \mu f[\frac{N_1}{N}V(g_1^{k^0}) + \frac{N_2}{N}V(T - g_1^{k^0})]$, and $F(\Delta)$ becomes $F(\Delta^{\mathcal{T}})$, in which case solving (40) is equivalent to solving (27).[14]

[14] That is, the problem becomes $\max_{g_1^A, g_1^B} F(\Delta^{\mathcal{T}})N_1V(g_1^A) + (1 - F(\Delta^{\mathcal{T}}))N_1V(g_1^B)$ $- \Big[\frac{1}{(1-\mu)e}\Big]\Big[(\overline{U^A} + \overline{U^B}) - \mu f[\frac{N_1}{N}(V(g_1^A) + V(g_1^B)) + \frac{N_2}{N}(V(T - g_1^A) + V(T - g_1^B))]\Big] - 2T$, whose solution is the same as that of (27).

To obtain the equilibrium when the constraints are slack and, thus, theft occurs, I derive the Kuhn-Tucker conditions from (40):[15]

$$N_1 F(\Delta) V'(g_1^{A^*}) + 1 - \lambda^{A^*} - \frac{\partial \Delta}{\partial g_1^{A^*}} \Big[N_1 \frac{\partial F}{\partial \Delta} [V(g_1^{B^*}) - V(g_1^{A^*})] - \frac{1}{(1-\mu)e} \Big] = 0 \tag{41}$$

$$1 - \lambda^{A^*} - \frac{\partial \Delta}{\partial g_2^{A^*}} \Big[N_1 \frac{\partial F}{\partial \Delta} [V(g_1^{B^*}) - V(g_1^{A^*})] - \frac{1}{(1-\mu)e} \Big] = 0 \tag{42}$$

$$N_1 (1 - F(\Delta)) V'(g_1^{B^*}) - 1 - \lambda^{B^*} - \frac{\partial \Delta}{\partial g_1^{B^*}} \Big[N_1 \frac{\partial F}{\partial \Delta} [V(g_1^{B^*}) - V(g_1^{A^*})] + \frac{1}{(1-\mu)e} \Big] = 0 \tag{43}$$

$$-1 - \lambda^{B^*} - \frac{\partial \Delta}{\partial g_2^{B^*}} \Big[N_1 \frac{\partial F}{\partial \Delta} [V(g_1^{B^*}) - V(g_1^{A^*})] + \frac{1}{(1-\mu)e} \Big] = 0 \tag{44}$$

$$\lambda^{A^*} (g_1^{A^*} + g_2^{A^*} - T) = 0 \tag{45}$$

$$\lambda^{B^*} (g_1^{B^*} + g_2^{B^*} - T) = 0, \tag{46}$$

where λ^{k^*} are the Lagrange multipliers.

Imposing $\lambda^{A^*} = 0$, (45) implies that $g_1^{A^*} + g_2^{A^*} < T$ while (41) and (42) imply that (i) $N_1 = (\frac{\partial g_2^{A^*}}{\partial g_1^{A^*}} - 1)(\frac{1}{V'(g_1^{A^*})F(\Delta)})$, where $\frac{\partial g_2^{A^*}}{\partial g_1^{A^*}} = \frac{\partial \Delta}{\partial g_1^{A^*}} \frac{\partial g_2^{A^*}}{\partial \Delta} = \frac{\mu f \frac{N_1}{N} V'(g_1^{A^*}) - (1-\mu)e}{\mu f \frac{N_2}{N} V'(g_2^{A^*}) - (1-\mu)e}$. Imposing $\lambda^{B^*} = 0$, (46) implies that $g_1^{B^*} + g_2^{B^*} < T$ while (43) and (44) imply that (ii) $N_1 = (1 - \frac{\partial g_2^{B^*}}{\partial g_1^{B^*}})(\frac{1}{V'(g_1^{B^*})(1-F(\Delta))})$, where $\frac{\partial g_2^{B^*}}{\partial g_1^{B^*}} = \frac{\partial \Delta}{\partial g_1^{B^*}} \frac{\partial g_2^{B^*}}{\partial \Delta} = \frac{-\mu f \frac{N_1}{N} V'(g_1^{B^*}) + (1-\mu)e}{-\mu f \frac{N_2}{N} V'(g_2^{B^*}) + (1-\mu)e}$. Equating (i) and (ii) and re-arranging, the equilibrium when theft occurs thus satisfies:

$$\frac{V'(g_1^{A^*})}{V'(g_1^{B^*})} = \frac{(1 - F(\Delta))(\frac{\partial g_2^{A^*}}{\partial g_1^{A^*}} - 1)}{F(\Delta)(1 - \frac{\partial g_2^{B^*}}{\partial g_1^{B^*}})} \tag{47}$$

The following results are readily obtained.

Proposition 3.8. *Both candidates offer to allocate some spending on each sector. That is,* $g_i^{k^*} > 0$ *for* $i = \{1, 2\}, k = \{A, B\}$.

Lemma 3.9. *For each* $k = \{A, B\}$, $\frac{\partial g_2^{k^*}}{\partial g_1^{k^*}} \neq |1|$. *If* $\frac{\partial g_2^{A^*}}{\partial g_1^{A^*}} \gtrless 1$, *then* $\frac{\partial g_2^{B^*}}{\partial g_1^{B^*}} \lessgtr 1$, *and vice-versa.*

[15] See appendix C for the derivation of (41) to (44).

Proposition 3.10. *Let* $w \equiv (\frac{\partial g_2^{A^*}}{\partial g_1^{A^*}} - 1)(\frac{\partial g_2^{A^*}}{\partial g_1^{A^*}} - \frac{\partial g_2^{B^*}}{\partial g_1^{B^*}})$. *If:*
(i) $F(\Delta) > w$, *then* $g_1^{A^*} < g_1^{B^*}$.
(ii) $F(\Delta) < w$, *then* $g_1^{A^*} > g_1^{B^*}$.
(iii) $F(\Delta) = w$, *then* $g_1^{A^*} = g_1^{B^*}$.

Proposition 3.11. *Neither candidate offers the socially optimal allocation. Both of them either overspend on group 1 or group 2.*

Note that when N_1 is large, it is easier to meet condition $\frac{V'(g_1^k)}{V'(g_2^k)} > \frac{V'(g_1^0)}{V'(T-g_1^0)} = \frac{N_2}{N_1}$ than $\frac{V'(g_1^k)}{V'(g_2^k)} < \frac{V'(g_1^0)}{V'(T-g_1^0)} = \frac{N_2}{N_1}$. Thus:

Corollary 3.12. *Both candidates are more likely to overspend on group 1 than on group 2 the larger the size of the former.*

To complete the analysis, I compare the equilibrium bribe offers to candidates A and B. Plugging $\overline{U_T^k}$ into (39) to get

$$b_T^{k^*} = \Big[\frac{1}{(1-\mu)e}\Big]\Big[\mu f[\frac{N_1}{N}(V(g_1^{k^0}) - V(g_1^{k^*})) + \frac{N_2}{N}(V(g_2^{k^0}) - V(g_2^{k^*}))]\Big] + g_1^{k^*} + g_2^{k^*} - (g_1^{k^0} + g_2^{k^0}), \quad (48)$$

one can take the difference:

$$\begin{aligned} b_T^{A^*} - b_T^{B^*} = \Big[\frac{1}{(1-\mu)e}\Big]\Big[\mu f[&\frac{N_1}{N}(V(g_1^{A^0}) - V(g_1^{A^*}) + V(g_1^{B^*}) - V(g_1^{B^0})) \\ &+\frac{N_2}{N}(V(g_2^{A^0}) - V(g_2^{A^*}) + V(g_2^{B^*}) - V(g_2^{B^0}))]\Big] \\ &+(g_1^{A^*} + g_2^{A^*}) - (g_1^{A^0} + g_2^{A^0}) + (g_1^{B^0} + g_2^{B^0}) - (g_1^{B^*} + g_2^{B^*}). \end{aligned} \quad (49)$$

It is not always the case that $g_1^{A^0} = g_1^{B^0}$ and $g_2^{A^0} = g_2^{B^0}$, since $g_i^{k^0}$ only requires $\frac{V'(g_1^{k^0})}{V'(g_2^{k^0})} = \frac{N_2}{N_1}$ for each $k = \{A, B\}$.[16] However, the latter also implies that if $g_1^{A^0} = g_1^{B^0}$, then $g_2^{A^0} = g_2^{B^0}$. The following result thus only needs to assume that there would be no difference in the candidates' behavior toward principal 1 if they were to reject the latter's offer, i.e. $g_1^{A^0} = g_1^{B^0}$.

Proposition 3.13. *Let* $g_1^{A^0} = g_1^{B^0}$. *Define* $x \equiv (T - g_1^{A^*} - g_2^{A^*}) - (T - g_1^{B^*} - g_2^{B^*}) = (g_1^{B^*} + g_2^{B^*}) - (g_1^{A^*} + g_2^{A^*})$, *the difference in stolen revenues from electing candidate A over B, and* $y \equiv$

[16]If k were to reject the bribe, she would choose $g_i^{k^0}$ by solving $\max_{g_1^k, g_2^k} U_T^k = \mu f[\frac{N_1}{N}V(g_1^{k^0}) + \frac{N_2}{N}V(g_2^{k^0})] - (1-\mu)e(g_1^{k^0} + g_2^{k^0})$, *s.t.* $g_1^{k^0} + g_2^{k^0} \le T$ when the constraint is slack, which yields $\frac{V'(g_1^{k^0})}{V'(g_2^{k^0})} = \frac{N_2}{N_1}$.

$[\frac{N_1}{N}V(g_1^{B^*}) + \frac{N_2}{N}V(g_2^{B^*})] - [\frac{N_1}{N}V(g_1^{A^*}) + \frac{N_2}{N}V(g_2^{A^*})]$, *the difference in social welfare from electing candidate B over A. If:*
(i) $x < [\frac{\mu f}{(1-\mu)e}]y$, *then* $b_T^{A^*} > b_T^{B^*}$.
(ii) $x > [\frac{\mu f}{(1-\mu)e}]y$, *then* $b_T^{A^*} < b_T^{B^*}$.
(iii) $x = [\frac{\mu f}{(1-\mu)e}]y$, *then* $b_T^{A^*} = b_T^{B^*}$.

That is, the bribes augment stolen revenues such that the less popular candidate can compensate for this disadvantage by using her combined rents to buy more votes and even out the competition.

4 Conclusion

In this paper, I have formally analyzed public-good spending by a rent-seeking government, where the rents can come from bribes and the theft of stolen revenues. To the best of my knowledge, the model I have proposed is the first to simultaneously consider these two sources of corruption. While there likely remain more insights to be obtained from such a model, I have been able to derive a number of important results. If the amount of government revenues are below some threshold demand for public spending, the government is constrained to spend all of the revenues to try to meet such demand. In this case, the "no-theft constraint" binds - the government cannot keep revenues for itself because it has to spend it all. However, the government can still obtain rents by spending the revenues in exchange for bribes. The implication is that when bribery is the only source of corruption, and the marginal value of the public goods from which bribes are extracted is sufficiently low, public-good spending and corruption are likely to be positively associated, which is why an increase in revenues is likely to increase both public-good spending and corruption.

If government revenues are larger than the threshold level required to meet the demand for public spending, then the government can steal the 'extra' revenues, and the no-theft constraint is non-binding. In this case, I find that any additional revenues above the threshold are *all* stolen. Above this threshold, no amount of bribes can induce the government to spend - it would rather keep all the additional revenues to herself. The reason is that if the government has the option to steal revenues, then for it to accept bribes, the principal/sector that offers bribes should offer an amount that is sufficiently high to cover two losses that the government incurs. First, given that the government cares about social welfare to some extent, it suffers a loss of social welfare from the misallocation of revenues, since the principal that bribes obtains a higher share of total spending than other principals that do not bribe. Second, spending the extra revenues means that the government foregoes rents from theft. Thus, when theft is possible, the bribes should be sufficiently high so as to compensate the government for having to misallocate revenues, and for foregoing theft. This means, however, that the marginal cost of preventing theft is larger than the government's marginal utility from stealing. Thus, all revenues above the threshold are stolen.

I have also shown that such rent-seeking behavior by the government is supportable even when the latter faces political competition. Political accountability is imperfect when the rents can be shared with citizens through, e.g. vote-buying.

The main limitation of the model is that it cannot ex ante determine whether or not theft is possible. The threshold demand for spending that has to be satisfied is itself endogenous in the model, since it can only be obtained by assuming that theft is *not* possible. Arguably, however, this captures the empirical reality of corruption in the allocation of public spending. If the true demand were easily determined, there would be no scope for theft, since one could easily calculate the amount of revenues that ought to remain in the government coffers. The fact that theft is endogenous with spending precisely makes theft possible.

Lastly, one can interpret the political resource curse through the model as a phenomenon in which (resource) revenues exceed the threshold demand for public goods, which allows the rent-seeking government to steal or appropriate the revenues at the expense of public-good spending.

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Appendices

A Ambiguous effect of revenues on social welfare

The following proves that the total social welfare losses may increase or decrease with revenues.

Let $\overline{T}$ denote the required threshold amount of spending, such that if actual revenues were below this, then no theft is possible. If actual revenues, T, were above the threshold, the agent obtains rents from bribes and stolen revenues, with the amount of bribes limited to the level associated with threshold spending $\overline{T}$, and the amount of stolen revenues equal to $T - \overline{T}$ (i.e. all revenues above the threshold are stolen).

Now consider two levels of actual revenues, T_1 and T_2, with $T_1 < \overline{T} < T_2$. If the social welfare loss at T_2 - denote as W_2, is greater (less) than that at T_1 - denote as W_1, then social welfare loss increases (decreases) with actual revenues. However, I now show that $W_2 \gtrless W_1$, in which case the social welfare loss may increase or decrease with revenues.

Since $T_2 > \overline{T}$, the total social welfare loss at T_2 is $W_2 = (T_2 - \overline{T}) + b_{\overline{T}}$, where the first term is the amount of stolen revenues, while the second term is the amount of bribes associated with $\overline{T}$. Since $T_1 < \overline{T}$, the social welfare loss at T_1 is $W_1 = b_1$, where b_1 is the amount of bribes associated with T_1.

Now, $W_2 \gtrless W_1$ if $(T_2 - \overline{T}) + b_{\overline{T}} \gtrless b_1$, or $(b_{\overline{T}} - b_1) \gtrless (T_2 - \overline{T})$. The RHS is positive, but the LHS is not always negative, which means that the social welfare loss at a relatively higher level of revenues, i.e. W_2, is not always higher than the loss at a lower level of revenues, i.e. W_1. More precisely, using equation (6): $(b_{\overline{T}} - b_1) = \frac{\lambda}{1-\lambda}\left[2(V(\frac{\overline{T}}{2}) - V(\frac{T_1}{2})) + (V(g_1^{T_1}) - V(g_1^{\overline{T}})) + (V(T - g_1^{T_1}) - V(T - g_1^{\overline{T}}))\right] \gtrless (T_2 - \overline{T})$.

B FOCs for $g_1^{k^*}$

To get equation (28), differentiate (27) with respect to $g_1^{A^*}$ and set to zero to get FOC:

$$\begin{aligned} N_1\Big[\frac{\partial F}{\partial \Delta^{\mathcal{T}}}\frac{\partial \Delta^{\mathcal{T}}}{\partial g_1^{A^*}}V(g_1^{A^*}) + F(\Delta^{\mathcal{T}})V'(g_1^{A^*})\Big] - N_1\frac{\partial F}{\partial \Delta^{\mathcal{T}}}\frac{\partial \Delta^{\mathcal{T}}}{\partial g_1^{A^*}}V(g_1^{B^*}) \\ +\Big[\frac{\mu f}{(1-\mu)e}\Big]\Big[\frac{N_1}{N}V'(g_1^{A^*}) - \frac{N_2}{N}V'(T - g_1^{A^*})\Big] = 0. \end{aligned} \tag{50}$$

Letting $g_2^{k^*} = T - g_1^{k^*}$ in (24), one can get

$$\frac{\partial \Delta^{\mathcal{T}}}{\partial g_1^{A^*}} = \mu f\Big[\frac{N_1}{N}V'(g_1^{A^*}) - \frac{N_2}{N}V'(T - g_1^{A^*})\Big]. \tag{51}$$

One can then write (50) as

$$N_1 F(\Delta^{\not T}) V'(g_1^{A^*}) = \frac{\partial \Delta^{\not T}}{\partial g_1^{A^*}} \Big[N_1 \frac{\partial F}{\partial \Delta^{\not T}} [V(g_1^{B^*}) - V(g_1^{A^*})] - \frac{1}{(1-\mu)e} \Big]. \tag{52}$$

Defining $x \equiv \frac{N_1 F(\Delta^{\not T})}{N_1 \frac{\partial F}{\partial \Delta^{\not T}} [V(g_1^{B^*}) - V(g_1^{A^*})] - [\frac{1}{(1-\mu)e}]}$, (52) becomes

$$x = \frac{\partial \Delta^{\not T}}{\partial g_1^{A^*}} \frac{1}{V'(g_1^{A^*})}. \tag{53}$$

Finally, writing out $\frac{\partial \Delta^{\not T}}{\partial g_1^{A^*}}$ in (53) using (51) and re-arranging give FOC (28).

FOC (29) can be similarly obtained. Differentiating (27) with respect to $g_1^{B^*}$ and setting to zero give

$$\begin{aligned} N_1 \frac{\partial F}{\partial \Delta^{\not T}} \frac{\partial \Delta^{\not T}}{\partial g_1^{B^*}} V(g_1^{A^*}) + N_1 V'(g_1^{B^*}) - N_1 \Big[\frac{\partial F}{\partial \Delta^{\not T}} \frac{\partial \Delta^{\not T}}{\partial g_1^{B^*}} V(g_1^{B^*}) + F(\Delta^{\not T}) V'(g_1^{B^*}) \Big] \\ + \Big[\frac{\mu f}{(1-\mu)e} \Big] \Big[\frac{N_1}{N} V'(g_1^{B^*}) - \frac{N_2}{N} V'(T - g_1^{B^*}) \Big] = 0. \end{aligned} \tag{54}$$

With

$$\frac{\partial \Delta^{\not T}}{\partial g_1^{B^*}} = -\mu f \Big[\frac{N_1}{N} V'(g_1^{B^*}) - \frac{N_2}{N} V'(T - g_1^{B^*}) \Big], \tag{55}$$

one can write (54) as

$$N_1 (1 - F(\Delta^{\not T})) V'(g_1^{B^*}) = \frac{\partial \Delta^{\not T}}{\partial g_1^{B^*}} \Big[N_1 \frac{\partial F}{\partial \Delta^{\not T}} [V(g_1^{B^*}) - V(g_1^{A^*})] + \frac{1}{(1-\mu)e} \Big]. \tag{56}$$

Defining $y \equiv \frac{N_1 (1 - F(\Delta^{\not T}))}{N_1 \frac{\partial F}{\partial \Delta^{\not T}} [V(g_1^{B^*}) - V(g_1^{A^*})] + [\frac{1}{(1-\mu)e}]}$, (56) becomes

$$y = \frac{\partial \Delta^{\not T}}{\partial g_1^{B^*}} \frac{1}{V'(g_1^{B^*})}. \tag{57}$$

Finally, writing out $\frac{\partial \Delta^{\not T}}{\partial g_1^{B^*}}$ and re-arranging give FOC (29).

C Kuhn-Tucker conditions for $g_i^{k^*}$

To get (41), get the derivative of the Lagrangian with respect to g_1^A and set to zero:

$$N_1 \Big[\frac{\partial F}{\partial \Delta} \frac{\partial \Delta}{\partial g_1^{A^*}} V(g_1^{A^*}) + F(\Delta) V'(g_1^{A^*}) \Big] - N_1 \frac{\partial F}{\partial \Delta} \frac{\partial \Delta}{\partial g_1^{A^*}} V(g_1^{B^*}) + \frac{\mu f}{(1-\mu)e} \frac{N_1}{N} V'(g_1^{A^*}) - \lambda^{A^*} = 0. \tag{58}$$

Using the fact that

$$\frac{\partial \Delta}{\partial g_1^{A^*}} = \mu f \frac{N_1}{N} V'(g_1^{A^*}) - (1-\mu)e \tag{59}$$

and re-arranging give (41).

To get (42), get the derivative of the Lagrangian with respect to g_2^A and set to zero:

$$N_1 V(g_1^{A^*}) \frac{\partial F}{\partial \Delta} \frac{\partial \Delta}{\partial g_2^{A^*}} - N_1 V(g_1^{B^*}) \frac{\partial F}{\partial \Delta} \frac{\partial \Delta}{\partial g_2^{A^*}} + \frac{\mu f}{(1-\mu)e} \frac{N_2}{N} V'(g_2^{A^*}) - \lambda^{A^*} = 0 \tag{60}$$

which, with

$$\frac{\partial \Delta}{\partial g_2^{A^*}} = \mu f \frac{N_2}{N} V'(g_2^{A^*}) - (1-\mu)e, \tag{61}$$

gives (42).

To get (43), get the derivative of the Lagrangian with respect to g_1^B and set to zero:

$$N_1 \frac{\partial F}{\partial \Delta} \frac{\partial \Delta}{\partial g_1^{B^*}} V(g_1^{A^*}) + N_1 V'(g_1^{B^*}) - N_1 \left[\frac{\partial F}{\partial \Delta} \frac{\partial \Delta}{\partial g_1^{B^*}} V(g_1^{B^*}) + F(\Delta) V'(g_1^{B^*}) \right] + \frac{\mu f}{(1-\mu)e} \frac{N_1}{N} V'(g_1^{B^*}) - \lambda^{B^*} = 0 \tag{62}$$

which, with

$$\frac{\partial \Delta}{\partial g_1^{B^*}} = -\mu f \frac{N_1}{N} V'(g_1^{B^*}) + (1-\mu)e, \tag{63}$$

gives (43).

To get (44), get the derivative of the Lagrangian with respect to g_2^B and set to zero:

$$N_1 \frac{\partial F}{\partial \Delta} \frac{\partial \Delta}{\partial g_2^{B^*}} V(g_1^{A^*}) - N_1 \frac{\partial F}{\partial \Delta} \frac{\partial \Delta}{\partial g_2^{B^*}} V(g_1^{B^*}) + \frac{\mu f}{(1-\mu)e} \frac{N_2}{N} V'(g_2^{B^*}) - \lambda^{B^*} = 0 \tag{64}$$

which, with

$$\frac{\partial \Delta}{\partial g_2^{B^*}} = -\mu f \frac{N_2}{N} V'(g_2^{B^*}) + (1-\mu)e, \tag{65}$$

gives (44).

Lastly, (45) and (46) are standard complementary-slackness conditions.

D Proofs

Proposition 2.1

Proof. By the implicit function theorem, $\frac{dg_1^*}{dT} = -\frac{\partial F/\partial T}{\partial F/\partial g_1^*} = \frac{\lambda V''(T-g_1^*)}{V''(g_1^*)+\lambda V''(T-g_1^*)} > 0$. Since $g_1^* + g_2^* = T$, then $\frac{dg_1^*}{dT} + \frac{dg_2^*}{dT} = 1$, which implies that $\frac{dg_2^*}{dT} = \frac{V''(g_1^*)}{V''(g_1^*)+\lambda V''(T-g_1^*)}$. Thus, (a) and (b) are obtained by comparing $\frac{\lambda V''(T-g_1^*)}{V''(g_1^*)+\lambda V''(T-g_1^*)}$ with $\frac{\lambda V''(g_1^*)}{V''(g_1^*)+\lambda V''(T-g_1^*)}$ or, simplifying, λ with $\frac{V''(g_1^*)}{V'(T-g_1^*)}$. ■

Remark Note that the second-order condition (SOC) for a maximum is $V''(g_1^*) + \lambda V''(T - g_1^*) < 0$, which is met since $V''(\cdot) < 0$. The SOC does not restrict the relative magnitudes of $V''(g_1^*)$ and $V''(T - g_1^*)$, as it only implies that $\lambda > \frac{V''(g_1^*)}{-V''(T-g_1^*)}$, which always holds, i.e. in both cases (a) and (b).

One can also show that $\frac{dg_1^*}{dT} \gtrless \frac{dg_2^*}{dT}$ by demonstrating that $\frac{dg_1^*}{dT} \gtrless \frac{1}{2}$. Since $g_1^* + g_2^* = T$, one can write $g_1^* = \alpha T$ and $g_2 = (1-\alpha)T$. With $g_1^* > g_2^*$, it must be that $\alpha \in (\frac{1}{2}, 1)$. Thus, if α is a constant, $\frac{dg_1^*}{dT} = \alpha > \frac{1}{2}$. However, more generally, $g_1^* = \alpha(\lambda, T)T$, with $\alpha(\lambda, T) > \frac{1}{2}$, in which case $\frac{dg_1^*}{dT} = \frac{\partial\alpha(\lambda,T)}{\partial T}T + \alpha(\lambda, T)$. Thus, $\frac{dg_1^*}{dT} \gtrless \frac{1}{2}$, since $\frac{1}{2} < \alpha(\lambda, T) \lesseqgtr \frac{1}{2} - \frac{\partial\alpha(\lambda,T)}{\partial T}$ if $\frac{\partial\alpha(\lambda,T)}{\partial T} \neq 0$.

A special case of $g_1^* = \alpha(\lambda, T)T$ is $g_1^* = f(\lambda)T^n$, where $f(\lambda) > 0$, and either $n \in (0, 1)$ or $n > 1$. In this case, $\frac{dg_1^*}{dT} = f(\lambda)(nT^{n-1}) \gtrless \frac{1}{2}$.[17] For an example in which $n \in (0, 1)$, suppose $g_1^* = \frac{\sqrt{T}}{1+\lambda}$, which implies $g_2^* = \frac{(1+\lambda)T-\sqrt{T}}{1+\lambda}$. Then $\frac{dg_1^*}{dT} = \frac{1}{2(1+\lambda)\sqrt{T}} \gtrless \frac{1}{2}$, since $\frac{1}{\sqrt{T}} \gtrless 1 + \lambda$. An example in which $n > 1$ is $g_1^* = \frac{T^2}{1+\lambda}$, which implies $g_2^* = \frac{(1+\lambda)T-T^2}{1+\lambda}$, and where $T \in (0, (1 + \lambda))$. In this case, $\frac{dg_1^*}{dT} = \frac{2T}{1+\lambda} \gtrless \frac{1}{2}$, since $T \gtrless \frac{1+\lambda}{4}$. ■

Proposition 2.2

Proof. Differentiating (5) with respect to T gives $\frac{\partial b^*}{\partial T} = \frac{\lambda}{1-\lambda}\Big[V'(\frac{T}{2}) - V'(g_1^*)\frac{dg_1^*}{dT} - V'(T-g_1^*)\big(1-\frac{dg_1^*}{dT}\big)\Big]$, which is greater than zero if $V'(\frac{T}{2}) > V'(g_1^*)\frac{dg_1^*}{dT} + V'(T - g_1^*)\big(1 - \frac{dg_1^*}{dT}\big)$ or, rearranging, $\frac{dg_1^*}{dT} < \frac{V'(\frac{T}{2})-V'(T-g_1^*)}{V'(g_1^*)-V'(T-g_1^*)}$. Items (b) and (c) directly follow. ■

Proposition 2.3

Proof. Applying the implicit function theorem to the system of equations (9), (10), (11), we know that necessary for g_1^*, g_2^*, γ^* to exist is that the inverse of

$$A = \begin{pmatrix} V''(g_1^*) & \frac{\lambda}{1-\lambda}V''(g_2^*) & -1 \\ 0 & \frac{\lambda}{1-\lambda}V''(g_2^*) & -1 \\ \gamma & \gamma & g_1^* + g_2^* - T \end{pmatrix} \tag{66}$$

exists or, equivalently, that the determinant of A is non-zero. Note that $\det A = V''(g_1^*)[\frac{\lambda}{1-\lambda}V''(g_1^*)(g_1^* + g_2^* - T) + \gamma^*]$. If theft occurs in equilibrium, then $g_1^* + g_2^* - T < 0$, which implies (from equation (11)) that $\gamma^* = 0$. Imposing $\gamma^* = 0$, $\det A = V''(g_1^*)[\frac{\lambda}{1-\lambda}V''(g_2^*)(g_1^* + g_2^* - T)]$, which is less than zero, unless $g_1^* = 0$ or $g_2^* = 0$ in which case $\det A = 0$. Thus, if sufficiency conditions are met such that g_1^*, g_2^*, γ^* exist, it must be that when theft occurs such that $\gamma^* = 0$, some revenues are allocated to both principals, i.e. $g_1^*, g_2^* > 0$. ■

[17] Note that n is non-negative since if $n < 0$, then $g_1^* = f(\lambda)\frac{1}{T^n}$, which implies $\frac{dg_1^*}{dT} = f(\lambda)\Big(\frac{-nT^{n-1}}{T^{2n}}\Big) < 0$. This is not possible, as (the proof of) Proposition 2.1 has shown that $\frac{dg_1^*}{dT} > 0$.

Proposition 2.4

Proof. Applying Cramer's rule,

$$\frac{dg_1^*}{dT} = -\frac{1}{\det A}\det\begin{pmatrix} V'(g_1^*)\frac{dg_1^*}{dT}\frac{1}{1-\lambda} - \frac{d\gamma^*}{dT} & 0 & -1 \\ V'(g_2^*)\frac{dg_2^*}{dT}\frac{1}{1-\lambda} - \frac{d\gamma^*}{dT} & \frac{\lambda}{1-\lambda}V''(g_2^*) & -1 \\ (g_1^* + g_2^* - T)\frac{d\gamma^*}{dT} - \gamma^* & \gamma^* & g_1^* + g_2^* - T \end{pmatrix}.$$

Now if theft occurs in equilibrium, $g_1^* + g_2^* < T$, which by equation (11) implies $\gamma^* = 0$ and, hence, $\frac{d\gamma^*}{dT}$. Imposing $\gamma^* = 0$ and $\frac{d\gamma^*}{dT}$ gives $\frac{dg_1^*}{dT} = -\frac{1}{\det A}[(g_1^* + g_2^* - T)V'(g_1^*)\frac{dg_1^*}{dT}\frac{1}{1-\lambda}V''(g_2^*)]$ or, simplifying, $\frac{dg_1^*}{dT} = 0$. Analogously,

$$\frac{dg_2^*}{dT} = -\frac{1}{\det A}\det\begin{pmatrix} V''(g_1^*)\frac{1}{1-\lambda} & V'(g_1^*)\frac{dg_1^*}{dT}\frac{1}{1-\lambda} - \frac{d\gamma^*}{dT} & -1 \\ 0 & V'(g_2^*)\frac{dg_2^*}{dT}\frac{1}{1-\lambda} - \frac{d\gamma^*}{dT} & -1 \\ \gamma^* & (g_1^* + g_2^* - T)\frac{d\gamma^*}{dT} - \gamma^* & g_1^* + g_2^* - T \end{pmatrix}.$$

Imposing $\gamma^* = 0$ and $\frac{d\gamma^*}{dT} = 0$ gives $\frac{dg_2^*}{dT} = -\frac{1}{\det A}[(g_1^* + g_2^* - T)V'(g_2^*)\frac{dg_2^*}{dT}\frac{1}{1-\lambda}V''(g_1^*)]$ or, simplifying, $\frac{dg_2^*}{dT} = 0$. ∎

Lemma 2.5

I first show that $g_1^0, g_2^0 > 0$ by applying the implicit function theorem to the system of equations (13), (14) and (15). That is, it is necessary that $\det B = \begin{pmatrix} \lambda V''(g_1^0) & 0 & -1 \\ 0 & \lambda V''(g_2^0) & -1 \\ \gamma^0 & \gamma^0 & g_1^0 + g_2^0 - T \end{pmatrix}$ is non-zero. Imposing $\gamma^0, \frac{d\gamma^0}{dT} = 0$ and evaluating, $\det B = (g_1^0 + g_2^0 - T)\lambda V''(g_1^0)\lambda V''(g_2^0)$ which is less than zero, unless $g_1^0 = 0$ or $g_2^0 = 0$. That is, assuming sufficiency conditions are met such that g_1^0, g_2^0, γ^0 exist, some spending is still allocated, i.e. $g_1^0, g_2^0 > 0$ even when theft occurs (i.e. $\gamma^0 = 0$).

To prove Lemma 2.5:

Proof. The proof is similar to the proof of Proposition 2.4. Applying Cramer's rule,

$$\frac{dg_1^0}{dT} = -\frac{1}{\det B}\det\begin{pmatrix} \lambda V''(g_1^0)\frac{dg_1^0}{dT} & 0 & -1 \\ \lambda V''(g_2^0)\frac{dg_2^0}{dT} & \lambda V''(g_2^*) & -1 \\ 0 & 0 & g_1^0 + g_2^0 - T \end{pmatrix}.$$

Now if theft occurs in equilibrium, $g_1^0 + g_2^0 < T$, which by equation (16) implies $\gamma^0 = 0$ and, hence, $\frac{d\gamma^0}{dT}$. Imposing $\gamma^0 = 0$ and $\frac{d\gamma^0}{dT} = 0$ gives $\frac{dg_1^0}{dT} = -\frac{1}{\det B}[(g_1^0 + g_2^0 - T)\lambda V''(g_1^0)\frac{dg_1^0}{dT}\lambda V''(g_2^0)$ or, simplifying, $\frac{dg_1^0}{dT} = 0$. Analogously,

$$\frac{dg_2^0}{dT} = -\frac{1}{\det B}\det\begin{pmatrix} \lambda V''(g_1^0) & \lambda V''(g_1^0)\frac{dg_1^0}{dT} & -1 \\ 0 & \lambda V''(g_2^0)\frac{dg_2^0}{dT} & -1 \\ 0 & 0 & g_1^0 + g_2^0 - T \end{pmatrix}.$$

Imposing $\gamma^0 = 0$ and $\frac{d\gamma^0}{dT} = 0$ gives $\frac{dg_2^0}{dT} = -\frac{1}{\det B}[(g_1^0 + g_2^0 - T)\lambda V''(g_1^0)\lambda V''(g_2^0)\frac{dg_2^0}{dT}]$ or, simplifying, $\frac{dg_2^0}{dT} = 0$. ∎

Proposition 2.6

Proof. Differentiating (16) with respect to T gives $\frac{\partial b^*}{\partial T} = \frac{\lambda}{1-\lambda}[V'(g_1^0)\frac{dg_1^0}{dT} + V'(g_2^0)\frac{dg_2^0}{dT} - V'(g_1^*)\frac{dg_1^*}{dT} - V'(g_2^*)\frac{dg_2^*}{dT}] - \frac{dg_1^0}{dT} - \frac{dg_2^0}{dT} + \frac{dg_1^*}{dT} + \frac{dg_2^*}{dT}$. By lemma 2.5, $\frac{dg_1^0}{dT}, \frac{dg_2^0}{dT} = 0$, and by Proposition 2.4, $\frac{dg_1^*}{dT}, \frac{dg_2^*}{dT} = 0$. Thus, $\frac{\partial b^*}{\partial T} = 0$. ■

Corollary 2.7

Proof. If theft occurs in equilibrium, then $R^* = T - g_1^* - g_2^* + b^*$. Differentiating with respect to T gives $\frac{\partial R^*}{\partial T} = 1 - \frac{dg_1^*}{dT} - \frac{dg_2^*}{dT} + \frac{\partial b^*}{\partial T}$, which is equal to 1 by Propositions 2.4 and 2.6. ■

Proposition 2.8

Proof. From corollary 2.7, $\frac{\partial R^*}{\partial T} = 1$. Note that $R^{\not{T}}$ is simply the amount of bribes when no theft is possible, and is thus given by equation (6). Thus, $\frac{\partial R^{\not{T}}}{\partial T}$ is equal to the expression for $\frac{\partial b^*}{\partial T}$ given in the proof of Proposition 2.2. Comparing such expression with 1 leads to items (a), (b), and (c). ■

Proposition 3.1

Proof. Note that if $\alpha^{k^*} > 0$, then $g_1^{k^*} < T$. For $\alpha^{A^*} > 0$, it must be that $\mu f \frac{N_1}{N} - x > 0$, and for $\alpha^{B^*} > 0$, it must be that $\mu f \frac{N_1}{N} + y > 0$. One can write (28) as (i) $\mu f \frac{N_2}{N} = (\mu f \frac{N_1}{N} - x)\frac{V'(g_1^{A*})}{V'(T-g_1^{A*})}$, and (29) as (ii) $\mu f \frac{N_2}{N} = (\mu f \frac{N_1}{N} + y)\frac{V'(g_1^{B*})}{V'(T-g_1^{B*})}$. Equating (i) and (ii) and re-arranging give $\frac{\mu f \frac{N_1}{N} - x}{\mu f \frac{N_1}{N} + y} = \frac{V'(T-g_1^{A*})}{V'(g_1^{A*})}\frac{V'(g_1^{B*})}{V'(T-g_1^{B*})}$. Since the RHS is non-negative, and $\mu f \frac{N_1}{N} \neq x$, $\mu f \frac{N_1}{N} \neq -y$, then $\mu f \frac{N_1}{N} - x > 0$ and $\mu f \frac{N_1}{N} + y > 0$. ■

Proposition 3.2

Proof. Note that $g_1^{A*} \gtrless g_1^{B*}$ if $\alpha^{A^*} \lesseqgtr \alpha^{B^*}$. In turn, $\alpha^{A^*} = \frac{\mu f \frac{N_2}{N}}{\mu f \frac{N_1}{N} - x} \lesseqgtr \frac{\mu f \frac{N_2}{N}}{\mu f \frac{N_1}{N} + y} = \alpha^{B^*}$ if $y \lesseqgtr -x$. Writing out the expressions for y and x, this condition becomes $\frac{N_1 \frac{\partial F}{\partial \Delta^{\not{T}}}[V(g_1^{B^*}) - V(g_1^{A^*})] - [\frac{1}{(1-\mu)e}]}{N_1 \frac{\partial F}{\partial \Delta^{\not{T}}}[V(g_1^{B^*}) - V(g_1^{A^*})] + [\frac{1}{(1-\mu)e}]} \lesseqgtr \frac{-F(\Delta^{\not{T}})}{1-F(\Delta^{\not{T}})}$. To simplify, let $a \equiv N_1 \frac{\partial F}{\partial \Delta^{\not{T}}}[V(g_1^{B^*}) - V(g_1^{A^*})]$ and $b \equiv \frac{1}{(1-\mu)e}$. Then the condition can be written as $\frac{a-b}{a+b} \lesseqgtr \frac{-F(\Delta^{\not{T}})}{1-F(\Delta^{\not{T}})}$, which simplifies to $F(\Delta^{\not{T}}) \lesseqgtr \frac{1}{2} - \frac{a}{2b}$, the RHS of which has been defined as z. (Result (iii) corresponds to $F(\Delta^{\not{T}}) = \frac{1}{2} - \frac{a}{2b}$). ■

Lemma 3.3

Proof. Writing out the expression for α^{A^*} and comparing with $\frac{N_2}{N_1}$ give: $\alpha^{A^*} = \frac{\mu f \frac{N_2}{N}}{\mu f \frac{N_1}{N} - x} \gtrless \frac{N_2}{N_1}$ or, simplifying, $x \gtrless 0$. (It follows that if $x = 0$, $\alpha^{A^*} = \frac{N_2}{N_1}$.) Similarly, $\alpha^{B^*} = \frac{\mu f \frac{N_2}{N}}{\mu f \frac{N_1}{N} + y} \gtrless \frac{N_2}{N_1}$ or, simplifying, $y \lesseqgtr 0$. (It follows that if $y = 0$, $\alpha^{B^*} = \frac{N_2}{N_1}$.) ■

Lemma 3.4

Proof. Note first that the numerators from the expressions for x and y are non-zero and positive since $F(\Delta^{\overline{T}}) \in (0, 1)$. Thus, whether $x, y \gtrless 0$ depend on their respective denominators. For $x \gtrless 0$, it must be that $V(g_1^{B^*}) - V(g_1^{A^*}) \gtrless \frac{1}{(1-\mu)eN_1 \frac{\partial F}{\partial \Delta^{\overline{T}}}} \equiv w$, where the RHS is greater than zero. Thus, when $g_1^{B^*} \leq g_1^{A^*}$, the LHS is less than or equal to zero, which implies $x < 0$. If $g_1^{B^*} > g_1^{A^*}$, then the LHS is greater than zero. In this case, one compares $V(g_1^{B^*}) - V(g_1^{A^*})$ with w. An analogous argument can be made to establish whether $y \gtrless 0$, which now requires $V(g_1^{A^*}) - V(g_1^{B^*}) \lesseqgtr w$. (Note that in this case, the LHS is less than or equal to zero when $g_1^{B^*} \geq g_1^{A^*}$, which implies $y > 0$.) ■

Proposition 3.5

Proof. To prove (i), note that Proposition 3.2 establishes that $g_1^{A^*} = g_1^{B^*}$ if $F(\Delta^{\overline{T}}) = z$. From Lemma 3.4, $x < 0$ and $y > 0$ if $g_1^{A^*} = g_1^{B^*}$. Finally, from Lemma 3.3, $\alpha^{A^*} < \frac{N_2}{N_1}$ if $x < 0$ and $\alpha^{B^*} < \frac{N_2}{N_1}$ if $y > 0$. Thus, both A and B attach higher weight to group 1's, than to group 2's utility, relative to $\frac{N_2}{N_1}$, which is the weight implies by the social optimum. Hence, $g_1^{A^*} = g_1^{B^*} > g_1^0$. Results (ii) and (iii) are analogously obtained using Proposition 3.2, and Lemmas 3.3 and 3.4. ■

Proposition 3.6

Proof. One can subtract $V(g_1^0)$ from both sides of the FOC for $g_1^{k^*}$ to get (i) $V(g_1^{k^*}) - V(g_1^0) = \alpha^{k^*} V(T - g_1^{k^*}) - V(g_1^0)$. Then, using the FOC for g_1^0, one can plug into the RHS of (ii) an expression for $V(g_1^0)$: (ii) $V(g_1^{k^*}) - V(g_1^0) = \alpha^{k^*} V(T - g_1^{k^*}) - \frac{N_2}{N_1} V(T - g_1^0)$. Finally, substituting the RHS of (ii) into the LHS of the condition for $b^{k^*} \geq 0$ gives $\alpha^{k^*} V(T - g_1^{k^*}) - \frac{N_2}{N_1} V(T - g_1^0) \leq \frac{N_2}{N_1}[V(T - g_1^{k^0}) - V(T - g_1^{k^*})]$, which reduces to $\frac{V(T-g_1^0)}{V(T-g_1^{k^*})} \geq \frac{\alpha^{k^*} N_1 + N_2}{2N_2}$. ■

Proposition 3.7

Proof. From equation (33), $b^{A^*} \gtrless b^{B^*}$ if $V(g_1^{B^*}) - V(g_1^{A^*}) \gtrless \frac{(1-\mu)e}{\mu f} \frac{N_2}{N_1}[V(T - g_1^{A^*}) - V(T - g_1^{B^*})]$. The LHS of the inequality is $\lesseqgtr 0$, while the RHS is $\gtrless 0$, when $g_1^{A^*} \gtrless g_1^{B^*}$. (Both the LHS and RHS are equal to zero when $g_1^{A^*} = g_1^{B^*}$, which implies $b^{A^*} = b^{B^*}$.) ■

Proposition 3.8

Proof. By assumption, $V'(g_i^{k^*}) \geq 0$, while equations (i) and (ii) underlying (47) require that $V'(g_i^{k^*}) \neq 0$. Thus, it must be that $V'(g_i^{k^*}) > 0$, which implies that $g_i^{k^*} > 0$. ■

Lemma 3.9

Proof. By proposition 3.8, the LHS of (47) is greater than zero. For the RHS to be greater than zero, $\frac{\partial g_2^{k^*}}{\partial g_1^{k^*}} \neq |1|$ and $\frac{\partial g_2^{A^*}}{\partial g_1^{A^*}} \gtrless 1 \iff \frac{\partial g_2^{B^*}}{\partial g_1^{B^*}} \lesseqgtr 1$. ■

Proposition 3.10

Proof. Note that $g_1^{A^*} \gtrless g_1^{B^*} \iff V'(g_1^{A^*}) \gtrless V'(g_1^{B^*})$, and $g_1^{A^*} = g_1^{B^*} \iff V'(g_1^{A^*}) = V'(g_1^{B^*})$. By (47), $V'(g_1^{A^*}) \gtrless V'(g_1^{B^*}) \iff (1 - F(\Delta))(\frac{\partial g_2^{A^*}}{\partial g_1^{A^*}} - 1) \gtrless F(\Delta)(1 - \frac{\partial g_2^{B^*}}{\partial g_1^{B^*}})$ or, re-arranging: $F(\Delta) \lesseqgtr (\frac{\partial g_2^{A^*}}{\partial g_1^{A^*}} - 1)(\frac{\partial g_2^{A^*}}{\partial g_1^{A^*}} - \frac{\partial g_2^{B^*}}{\partial g_1^{B^*}}) \equiv w$, while $V'(g_1^{A^*}) = V'(g_1^{B^*}) \iff F(\Delta) = w$. ■

Proposition 3.11

Proof. The socially optimal allocation, i.e. when there is no theft or bribery such that $\overline{U^k} = \mu f[\frac{N_1}{N}V(g_1^k) + \frac{N_2}{N}V(T - g_1^k)]$ is maximized, satisfies $\frac{V'(g_1^0)}{V'(T-g_1^0)} = \frac{N_2}{N_1}$. Now the allocation of each candidate is such that $\frac{V'(g_1^k)}{V'(g_2^k)} \gtrless \frac{V'(g_1^0)}{V'(T-g_1^0)}$. To see this, recall that $\frac{\partial g_2^{A^*}}{\partial g_1^{A^*}} = \frac{\partial \Delta}{\partial g_1^{A^*}}\frac{\partial g_2^{A^*}}{\partial \Delta} = \frac{\mu f \frac{N_1}{N}V'(g_1^{A^*}) - (1-\mu)e}{\mu f \frac{N_2}{N}V'(g_2^{A^*}) - (1-\mu)e}$. By Lemma 3.9, $\mu f \frac{N_1}{N}V'(g_1^{A^*}) - (1-\mu)e \gtrless \mu f \frac{N_2}{N}V'(g_2^{A^*}) - (1-\mu)e$ or, simplifying, $\frac{V'(g_1^A)}{V'(g_2^A)} \gtrless \frac{N_2}{N_1}$. Similarly, $\frac{\partial g_2^{B^*}}{\partial g_1^{B^*}} = \frac{\partial \Delta}{\partial g_1^{B^*}}\frac{\partial g_2^{B^*}}{\partial \Delta} = \frac{-\mu f \frac{N_1}{N}V'(g_1^{B^*}) + (1-\mu)e}{-\mu f \frac{N_2}{N}V'(g_2^{B^*}) + (1-\mu)e}$ and, by Lemma 3.9, $-\mu f \frac{N_1}{N}V'(g_1^{B^*}) + (1-\mu)e \lesseqgtr -\mu f \frac{N_2}{N}V'(g_2^{B^*}) + (1-\mu)e$ or $\frac{V'(g_1^B)}{V'(T-g_1^B)} \gtrless \frac{N_2}{N_1}$. Substituting in for $\frac{N_2}{N_1}$, we have that $\frac{V'(g_1^k)}{V'(g_2^k)} \gtrless \frac{V'(g_1^0)}{V'(T-g_1^0)}$ for each $k = \{A, B\}$. ■

Proposition 3.13

Proof. For $\frac{V'(g_1^{k^0})}{V'(g_2^{k^0})} = \frac{N_2}{N_1}$ to hold for each candidate A and B, it must be that $g_1^{A^0} = g_1^{B^0} \iff g_2^{A^0} = g_2^{B^0}$. In this case, (49) reduces to $\left[\frac{1}{(1-\mu)e}\right]\left[\mu f[\frac{N_1}{N}(V(g_1^{B^*}) - V(g_1^{A^*})) + \frac{N_2}{N}(V(g_2^{B^*}) - V(g_2^{A^*}))]\right] + g_1^{A^*} + g_2^{A^*} - (g_1^{B^*} + g_2^{B^*})$. Thus, $b_T^{A^*} \gtrless b_T^{B^*}$ if $\left[\frac{1}{(1-\mu)e}\right]\left[\mu f[\frac{N_1}{N}(V(g_1^{B^*}) - V(g_1^{A^*})) + \frac{N_2}{N}(V(g_2^{B^*}) - V(g_2^{A^*}))]\right] + g_1^{A^*} + g_2^{A^*} - (g_1^{B^*} + g_2^{B^*}) \gtrless 0$ or, re-arranging: $[(g_1^{B^*} + g_2^{B^*}) - (g_1^{A^*} + g_2^{A^*})] \lesseqgtr [\frac{\mu f}{(1-\mu)e}][\frac{N_1}{N}V(g_1^{B^*}) + \frac{N_2}{N}V(g_2^{B^*})] - [\frac{N_1}{N}v(g_1^{A^*}) + \frac{N_2}{N}V(g_2^{A^*})]$, or $x \lesseqgtr [\frac{\mu f}{(1-\mu)e}]y$. (Result (iii) immediately follows.) ■